A REAL TIME VISIBLE SURFACE ALGORITHM

Gary Scott Watkins

Utah University

Prepared for:

Rome Air Development Center
Advanced Research Projects Agency

June 1970

A real-time visible surface algorithm

GARY SCOTT WATKINS

UNIVERSITY OF UTAH

JUNE 1970
UTEC-CSc-70-101
COMPUTER SCIENCE, UNIVERSITY OF UTAH
SALT LAKE CITY, UTAH 84112

A REAL TIME VISIBLE SURFACE ALGORITHM

by

Gary Scott Watkins

June 1970

UTEC-CSc-70-101

This research was supported in part by the University of Utah Computer Science Division and the Advanced Research Projects Agency of the Department of Defense, monitored by Rome Air Development Center, Griffiss Air Force Base, New York 13440, under contract AF30(602)-4277. ARPA Order No. 829.

TABLE OF CONTENTS

ACKNOWLEDGMENTS			iii
LIST OF ILLUSTRATIONS			vi
ABSTRACT			viii
CHAPTER I		INTRODUCTION	1
	A.	Path of Edges Algorithms	1
	B.	Sample Space Algorithms	3
CHAPTER II		PRE-FRAME PROCESSING	7
CHAPTER III		VISIBLE SEGMENT GENERATOR	9
	A.	Segment Generator	9
	B.	Segment Eliminator	13
	C.	Depth Sorter	14
	D.	Sampling	14
	E.	Sample Space Generator	16
	F.	Depth Comparator	18
	G.	Segment Clipping	22
	H.	Decision Processor	28
	I.	Intersecting Segments	32
	J.	Building the Sample List	34
CHAPTER IV		FRAME-TO-FRAME COHERENCE	35
CHAPTER V		RELATIONSHIP WITH OTHER ALGORITHMS	36
CHAPTER VI		DEVELOPMENT OF THE VSG ALGORITHM	38
CHAPTER VII		TEST DATA	40
	A.	Objects	40

B.	Statistics	41
C.	Analysis	41
D.	Output Buffering	58
CHAPTER VIII	CONCLUSION	60
BIBLIOGRAPHY		64
APPENDIX I	LISTING OF PROGRAM	66
APPENDIX II	STATISTICS OF OBJECTS AND ALGORITHMS	88

LIST OF ILLUSTRATIONS

Figure 1	Cube Presenting Optical Illusion	2
Figure 2	Cube with Hidden Edges not Drawn	2
Figure 3	Classification of Algorithms	5
Figure 4	Description of Edge and Polygon Blocks	8
Figure 5	Segment Block	10
Figure 6	Segments	11
Figure 7	Packing of Polygon Segment List	15
Figure 8	Sampling Points	17
Figure 9	Sample Edges and Sample Points	19
Figure 10	VSG Flowchart	20
Figure 11	Two Segments on a Scan Line	23
Figure 12	Arithmetic Unit for Depth Comparator	24
Figure 13	Gating of a Single Quadrant	25
Figure 14	Clipping of Segments	27
Figure 15	Boxing of Segments	29
Figure 16	Elimination of Visible Box by Visible Segment	29
Figure 17	Subdivision	30
Figure 18	Three Potentially Visible Segments	31
Figure 19	Intersecting Segments	31
Figure 20	Intersecting Segments Clipped to X_{lclip} and X_{rclip}	33

Figure 21	Registers for Finding Intersection	33
Figure 22	Object 1: Penetration	42
Figure 23	Object 2: E-S	43
Figure 24	Object 3: Low Area	44
Figure 25	Object 4: Cubel	45
Figure 26	Object 5: Cube2	46
Figure 27	Object 6: Shapel	47
Figure 28	Object 7: Shape2	48
Figure 29	Object 8: Sheet	49
Figure 30	Object 9: Simplel	50
Figure 31	Object 10: Simple2	51
Figure 32	Statistics of the Penetration Object for the Six Algorithms	52
Figure 33	Statistics of VSG6 for the Ten Test Objects	53
Figure 34	Office Structure	61
Figure 35	Church	61
Figure 36	Rear View of Church with Randomly Colored Blocks	62
Figure 37	Apollo Command and Service Module	62
Figure 38	Tori	63
Figure 39	Randomly Colored Surface	63

ABSTRACT

With the increasing use of computer graphics, a need is growing for a processor capable of displaying solid objects. Environmental simulation and architectural modeling are only two areas that would benefit from such a display processor.

This dissertation describes an algorithm designed for such a processor, and a program for simulating the hardware processor. The hardware processor would be capable of generating pictures of fairly complicated objects at thirty frames per second. Statistics describing its simulated performance have been extracted and are reported within the dissertation.

CHAPTER I

INTRODUCTION

With the introduction of line-drawing displays, it was soon realized that displaying too much information detracted from the meaning and actually confused the picture. For instance, a single cube can create an optical illusion as shown in Figure 1. However, the optical illusion is removed if lines hidden by surfaces in front of them are not displayed (see Figure 2). A different approach could be taken. Instead of determining the hidden lines, an algorithm could find, color, and shade visible surfaces, thus presenting a more true to life picture. For the past several years, different algorithms have been developed for solving the hidden line or visible surface problem. The various algorithms can be classified into several groups.

A. Path of Edges Algorithms

Some solutions to the problem have been found by various methods of tracing along the edges of objects and noting which of the edges are wholly or partially visible. The resulting picture is then the display of the visible segments of edges. Algorithms based on this method have been developed by Roberts [1], Loutrel [2], and Appel [3]. This type of approach does not take into account the resolution of the display but solves the hidden line problem to

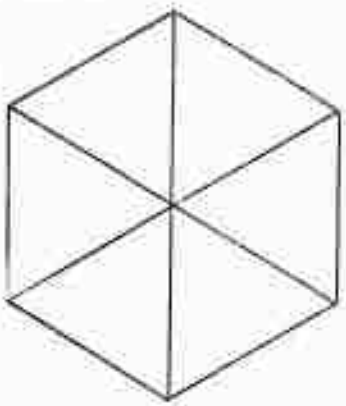


Figure 1

Cube Presenting Optical Illusion

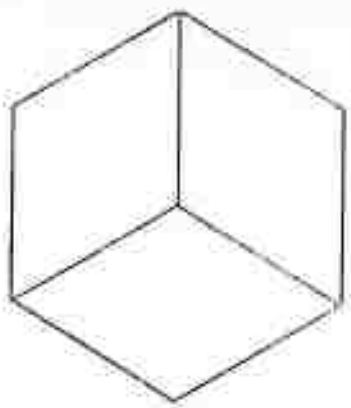


Figure 2

Cube with Hidden Edges not Drawn

the precision inherent in the object description.

B. Sample Space Algorithms

In 1967 a paper was presented by Wylie, Romney, Evans, and Erdahl [4]. One of the concepts discussed was initiated by Evans and introduced the concept of a sample space. The concept states that given an output device with resolution of R_x by R_y, one need only solve the hidden line problem at the discrete resolution points. The sample space can be thought of as taking the original object description in X, Y, Z, and mapping the object on to a two dimensional grid of resolution R_x by R_y. Of course, the Z information needs to be preserved in some form. When this is done, the object will exist only at discrete points in X and Y. The reasoning behind this was when a person views a picture he is physically limited by the resolution of the eye and the resolution of the display device. Hence, the hidden line problem need only be solved to the coarser resolution of the two.

In the algorithm, non-intersecting triangles were used as the object description. However, convex polygons could have been used with only small changes in the program. The algorithm used a scan line approach. That is, one Y raster line would be completely solved for visible triangles before the program proceeded to the next scan line. A method of sorting vertices of the triangles was developed by Wylie

and Romney so only triangles concerned with the current scan line were considered. On each scan line, triangle depths were compared only where edges of the triangles crossed the current scan line. Therefore, it was not necessary to do depth computations at all raster points. Later Romney [5] improved the sorting technique and added a "speedy" check to the program to take advantage of scan line-to-scan line coherence. This improved the speed by eliminating depth sorting as long as triangles entering on the scan line were ordered the same as the previous scan line.

Warnock [6] took a new approach but still kept the grid of resolution points. The object description was generalized by allowing polygons (convex or non-convex) which could intersect one another. Instead of the scan line approach, Warnock took an area of the picture and tried to "understand" it. If it was simple enough to "understand" he would display it, otherwise he subdivided the area into smaller areas. Eventually, a sub-area could be "understood" and displayed, or a sub-area would reach resolution whereupon it would be displayed without further analysis. This concept of subdividing large problems into smaller (and easier) problems is a "non-deterministic" algorithm.

After Warnock's algorithm was developed, Bouknight [7] took the scan line approach and generalized it to include general polygons which could intersect. Figure 3 shows a classification of the various algorithms.

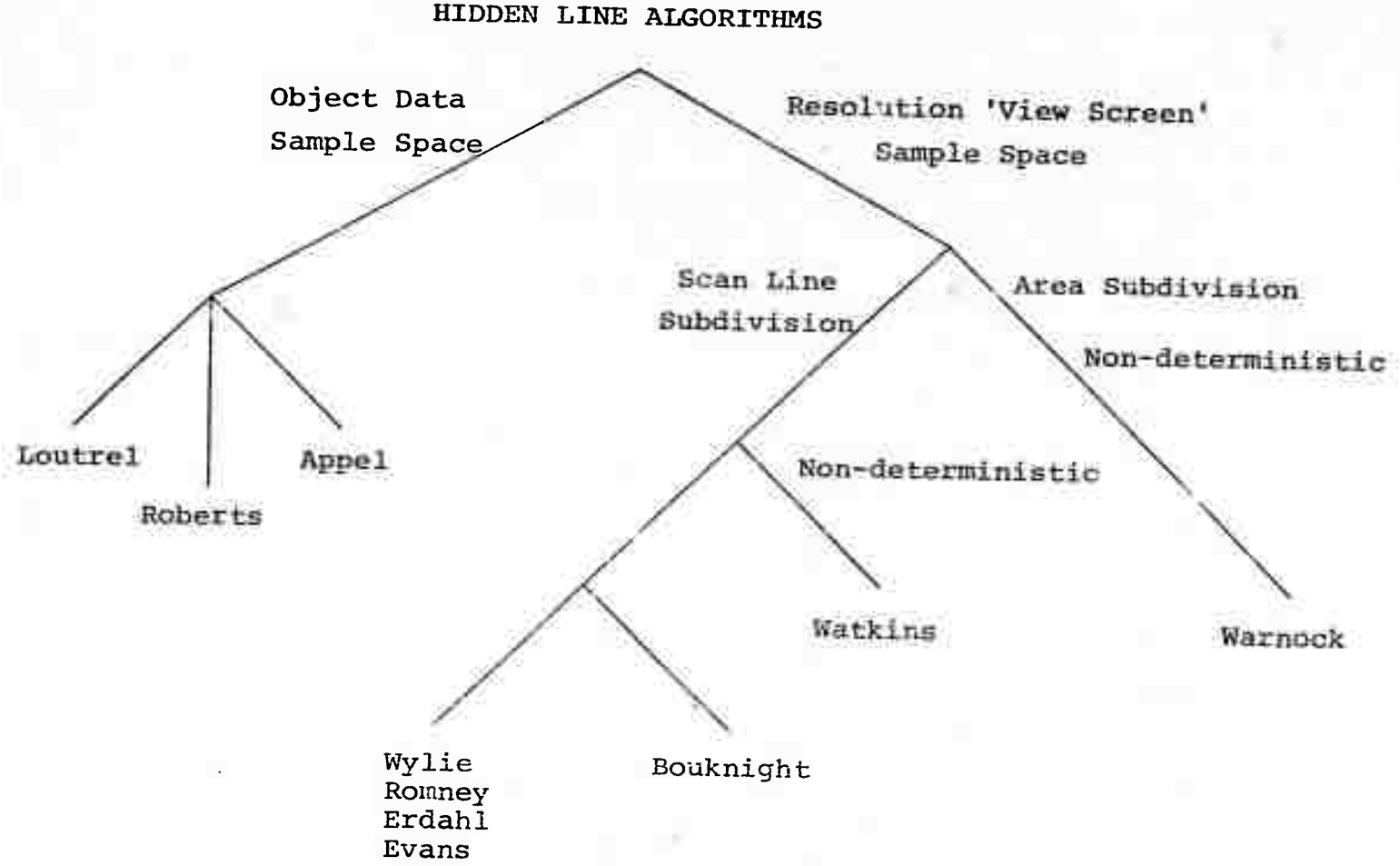


Figure 3
Classification of Algorithms

The new algorithm to be described is of the mapped sample space class, and it allows general polygons which can intersect. Key ideas used in this algorithm are: (1) Scan line-to-scan line coherence of pictures, and (2) an arithmetic unit for Z-depth sorting. Frame-to-frame coherence was not found valuable (in terms of increasing the speed of the program) for inclusion in this final algorithm.

The program implementing this algorithm is a simulation of hardware to generate visible segments of polygons on each scan line at real time speeds. Thus, the program is a Visible Segment Generator (VSG). The output of the VSG is given to a shader for displaying. The method of shading is very similar to that described by Romney [5] and Warnock [6].

CHAPTER II

PRE-FRAME PROCESSING

Before being accepted by the VSG, the object must be processed so that all translations, rotations, and perspective transformations have been applied. All polygons must be clipped at the boundaries of the viewing sample space. Since the scanning process proceeds from Y=1 to Y=512 (or to the Y-resolution value), the edges must be ordered in a list according to the minimum Y value (Y-min) of each edge. Horizontal edges need not be put in the list since the VSG will reject them. On any scan line the VSG can then immediately find which (if any) edges enter on that particular scan line. For each polygon, three fields are zeroed initially and reserved as sorting fields for the VSG. The formats for the edge block and polygon block are shown in Figure 4. The shading and color information will never be used by the VSG for computations. However, the VSG will pass the information to the shader for displaying if the object is visible.

A user that describes objects as closed polyhedra can double the speed of the processor if edge and polygon blocks are only created for polygons that face the viewer. This process was used on the test objects described in Chapter VII.

EDGE BLOCK

POINTER TO NEXT EDGE BLOCK
POINTER TO POLYGON BLOCK
Y - MAX
Y - MIN
X - BEGIN (ASSOCIATED WITH Y - MIN)
ΔX
Z - BEGIN (ASSOCIATED WITH Y - MIN)
ΔZ

POLYGON BLOCK

POINTER TO INITIAL SEGMENT ON POLYGON
POINTER TO NEXT CHANGING POLYGON
POLYGON ACTIVE BIT
SHADING AND COLORING INFORMATION

Figure 4

Description of Edge and Polygon Blocks

CHAPTER III

VISIBLE SEGMENT GENERATOR

The VSG can be broken into three separate processors: (1) Segment Generator, (2) Segment Eliminator, and (3) Depth Sorter.

A. Segment Generator (SG)

The format for a segment block is shown in Figure 5. A segment is defined as the continuous surface of a polygon which exists between two adjacent edges on a scan line. Thus in Figure 6, on scan line 'a' there are two segments, while on scan line 'b' these two segments of the polygon have merged into one segment. A segment block contains a description of the two bounding edges. The two Y-end values specify the Y scan lines when the edges exit from the picture. The X and Z values are stored along with the ΔZ and ΔX increments for each edge. Thus, when the program proceeds to the next scan line, the X and Z values are updated by adding the increments as in Equation 1.

$$Z \leftarrow Z+\Delta Z \quad ; \quad X \leftarrow X+\Delta X \tag{1}$$

The segment blocks are threaded together by four separate list structures:

1. The X-sort list contains all segments on the current scan line sorted with respect to the left edge of each segment. This list has both forward and backward

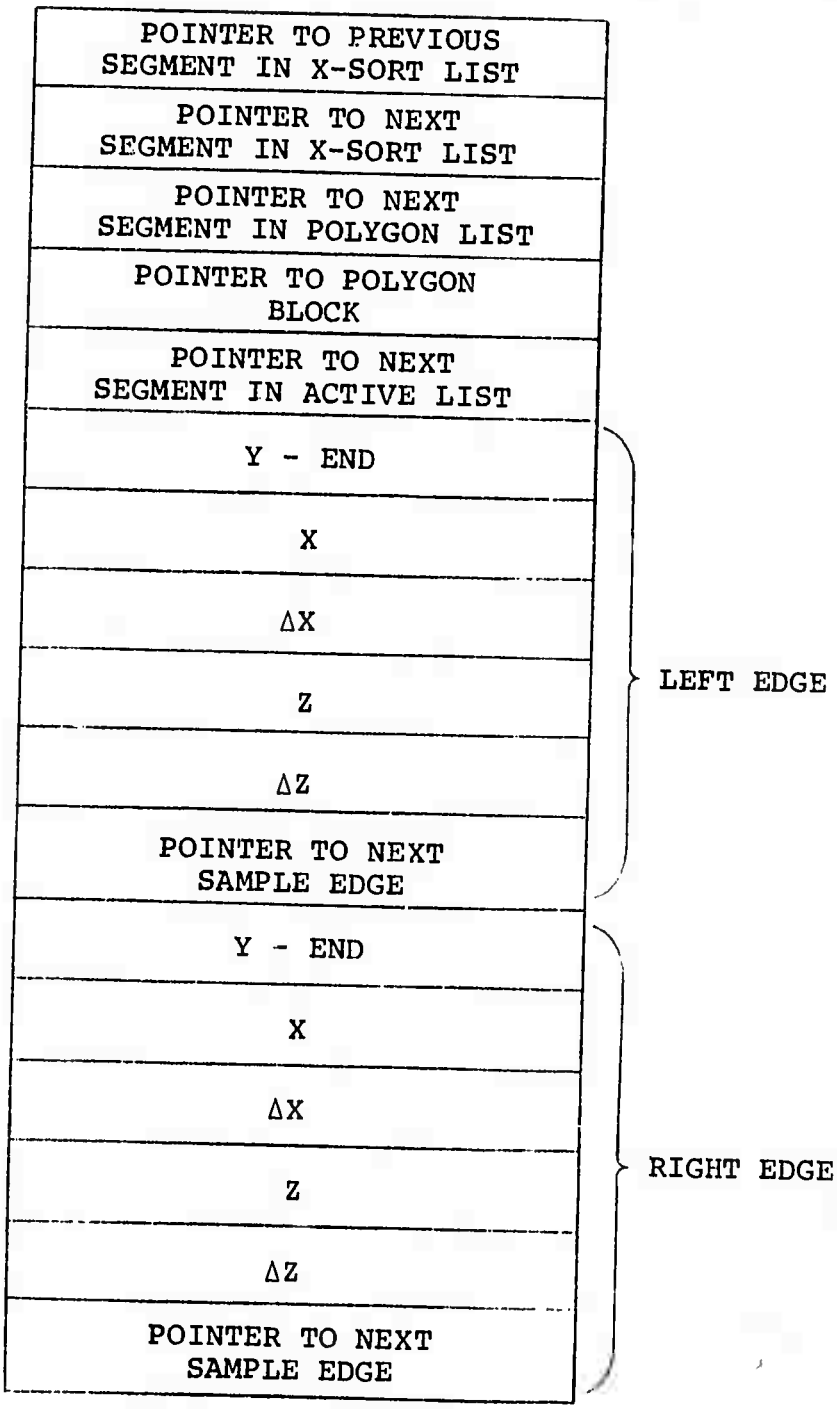


Figure 5

Segment Block

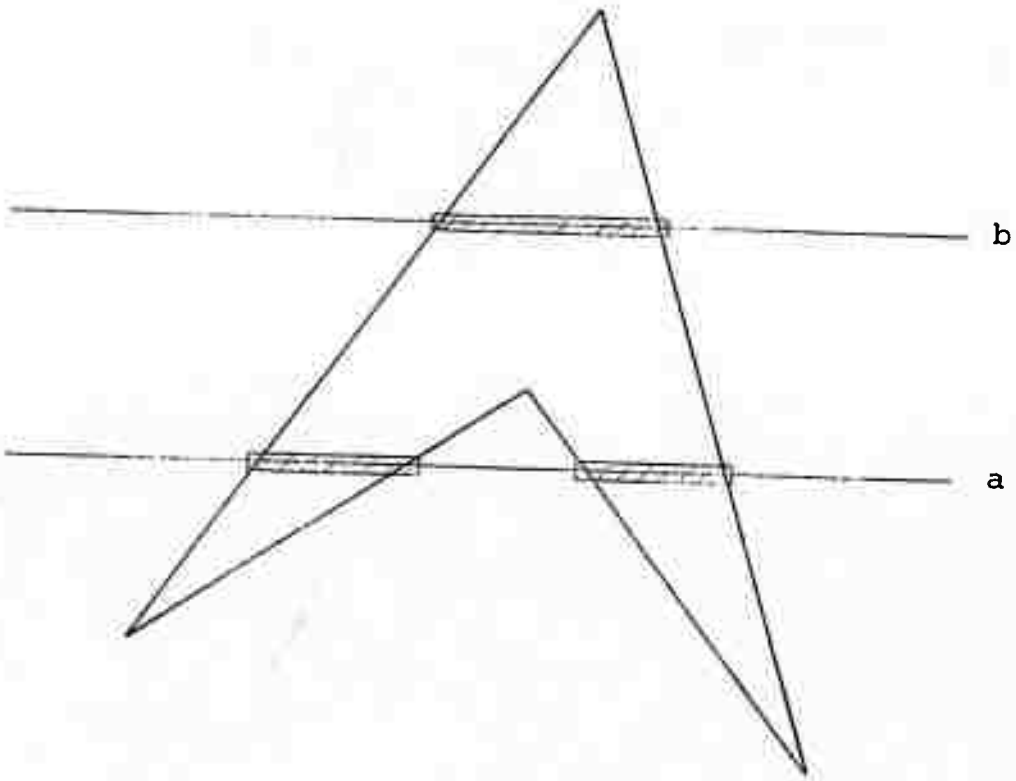


Figure 6
Segments

pointers.

2. Each polygon segments list contains an ordered set of all segments belonging to a particular polygon on a scan line. They are linked together, with the initial pointer (contained in the polygon block) pointing to the left most segment of the polygon.

3. The active segment list contains only segments of the X-sort list which exist in a specified range of X values. Section F of this chapter will give more detail of it.

4. The sample list is another sorted list that will be explained later.

The SG is checked on each scan line to see if any new edges enter the current scan line from the edge list. If there are no entering edges, control is passed to the segment eliminator. If edges do enter on a scan line, data from the edges is used to create a segment.

The polygon block associated with the incoming edge is checked to see if the active bit is set. Active designates whether or not the polygon is already in the list of changing polygons (polygons that have edges entering or exiting on the current scan line). If the polygon was not previously active, it is tagged as active and put in the list containing all changing polygons on this scan line.

Since an edge has only enough data for one half of a segment, an edge can be inserted into either the right or

left side of an empty segment. Because the program does not know whether an edge bounds the right or left side of a polygon, the algorithm may insert an edge into the wrong half of a segment. However, if this happens, the Segment Eliminator will do the necessary rearranging. The X value of the incoming edge is compared against the X values of segments in the polygon segments list until the appropriate location in the list is found for inserting the edge data.

After finding the correct location in the list, and if there is not an empty half of a segment block, the SG must get a block from free storage and insert it in the list at the correct location. Pointers to the segment block must also be inserted in the X-sort list in the correct location whenever data is stored in the left half of the block.

The preceding process is repeated for all edges that enter on the current scan line. Finally when no more edges enter, control is passed to the Segment Eliminator.

B. Segment Eliminator (SE)

The SE runs through the list of all changing polygons, and for each of the polygons it disconnects the polygon from the changing polygon list, and resets the active bit. It then proceeds through the list of segments attached to that polygon to determine if any data needs to be shifted from one segment block to another, or if any segment blocks can be returned to free storage. For example, in Figure 6 on scan line 'a' the polygon has two segments. Because the two

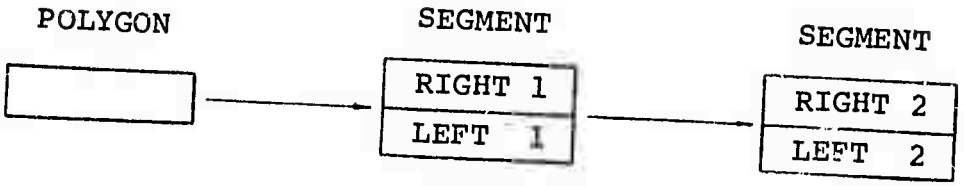
middle edges exit between scan lines 'a' and 'b,' this polygon will have been inserted into the list of changing polygons. The SE must then take the right edge data from the second segment block and insert that data into the right half of the first segment block. After this, the second segment block will be returned to free storage. Figure 7 gives a step-by-step illustration of what would happen if displaying the polygon in Figure 6. When all active polygons have been checked by the SE, control is passed to the Depth Sorter.

C. Depth Sorter (DS)

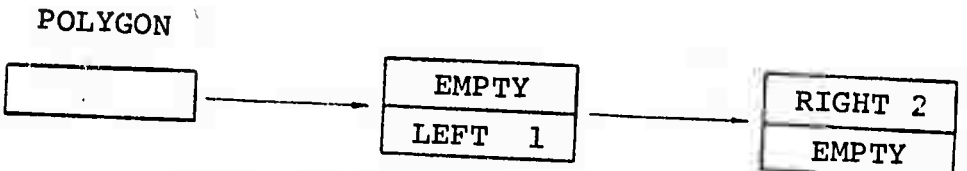
At this point the X-sort list contains all segment blocks on this scan line ordered with respect to the left edge of each scan line. While the SG and SE are concerned only with polygons that change on the current scan line, the DS is concerned with all polygons that exist on the scan line. Therefore, the list handling and memory referencing in this processor are extremely critical to the overall speed of the VSG.

D. Sampling

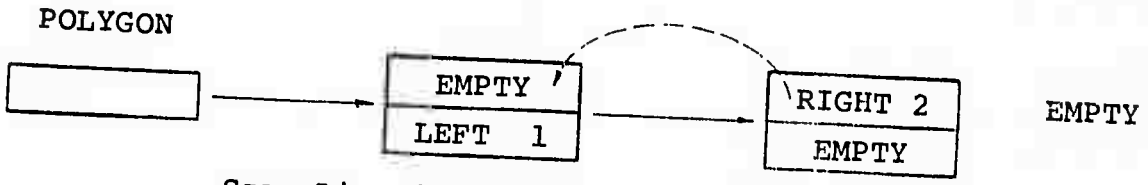
A critical factor in the speed of the algorithm is the number of points on the scan line where depths of polygons are sampled. The depth sorter is capable of determining at most a single visible segment for a restricted span of a scan line. Because of this, sampling must at least be done



Scan Line 'a'



Scan Line 'a' after Noticing Two Edges
Exit on this Scan Line



Scan Line 'b' Showing Data Transfer

Scan Line 'b' after Second Segment Block
Has Been Returned to Free Storage

Figure 7
Packing of Polygon Segment List

at the points of discontinuity (the visible edges). Scan line-to-scan line coherence usually allows the DS to find the visible segment by sampling only at the visible edges contained in the sample list. For the object in Figure 8, one notices the sampling points actually following the visible edges of the picture. Thus the speed of the algorithm will be more dependent on the visible complexity of the object than on the total object complexity.

The Depth Sorter can be subdivided into three separate processors: (1) The Sample Space Generator, (2) The Depth Comparator, and (3) The Decision Processor.

E. Sample Space Generator (SSG)

The SSG operates from the sample list. Essentially the list contains the sorted edges (each half of a segment block is an edge) which were visible on the previous scan line. The building of the Sample List was done on the previous scan line by the Decision Processor and will be discussed under that heading.

The left and right sides of the view screen are always implied sample edges. The scan process on a single scan line proceeds from left to right in X. Therefore, the left edge of the view screen becomes the initial left sample point. The X value of the first edge in the sample list then becomes the right sample point. This sample edge is then removed from the sample list. The portion on the scan line which exists between the left and right sample points

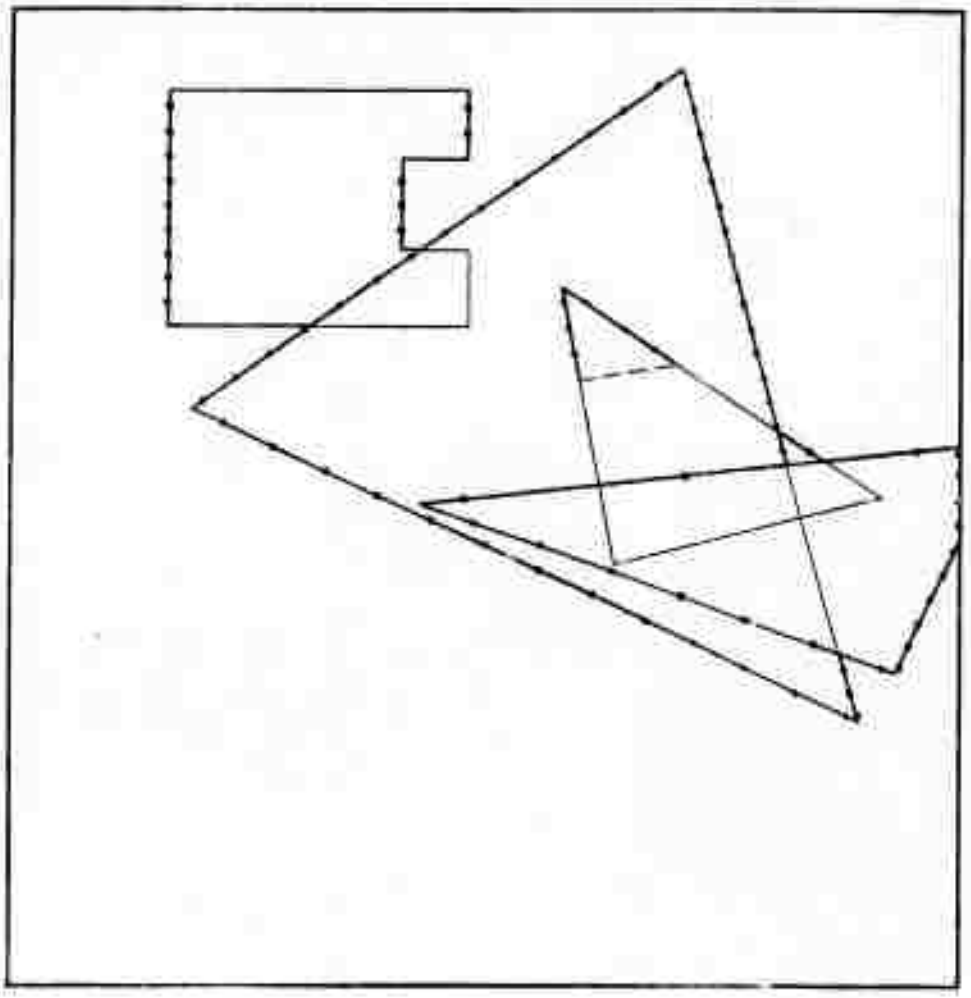


Figure 8
Sampling Points

is called a span.

Suppose in Figure 9 one found on scan line 'w' that edges B, C, and D were visible, and therefore they were put in the sample list with B at the first of the list. When the program proceeds to scan line 'w+1,' the current X value of edge A is initially set as the left sample point. Edge B then becomes the right sample point. Once a left and right sample point is found, control is passed to the Depth Sorter and Decision Processor. Finally, when the Decision Processor finishes its task, control is passed back to SSG. Now the right sample point 'b' becomes the left sample point. Edge C is read from the sample list, point 'c' becomes the right sample point, and the cycle begins again. The cycling process finally stops when the end of the scan line is reached whereupon control is passed back to the Segment Generator for the next scan line. A flow chart in Figure 10 shows the overall control of the system.

F. Depth Comparator (DC)

The DC takes all the segments from the X-sort list that exist between the left and right sample points and operates on them in the following manner: (1) The X and Z values are incremented to the values associated with the next scan line and stored back in the segment block. (2) If either of the edges of the segment exit on this scan line, the associated polygon is tagged as active and put in the changing polygon list. (3) The X value of the left edge is compared with the

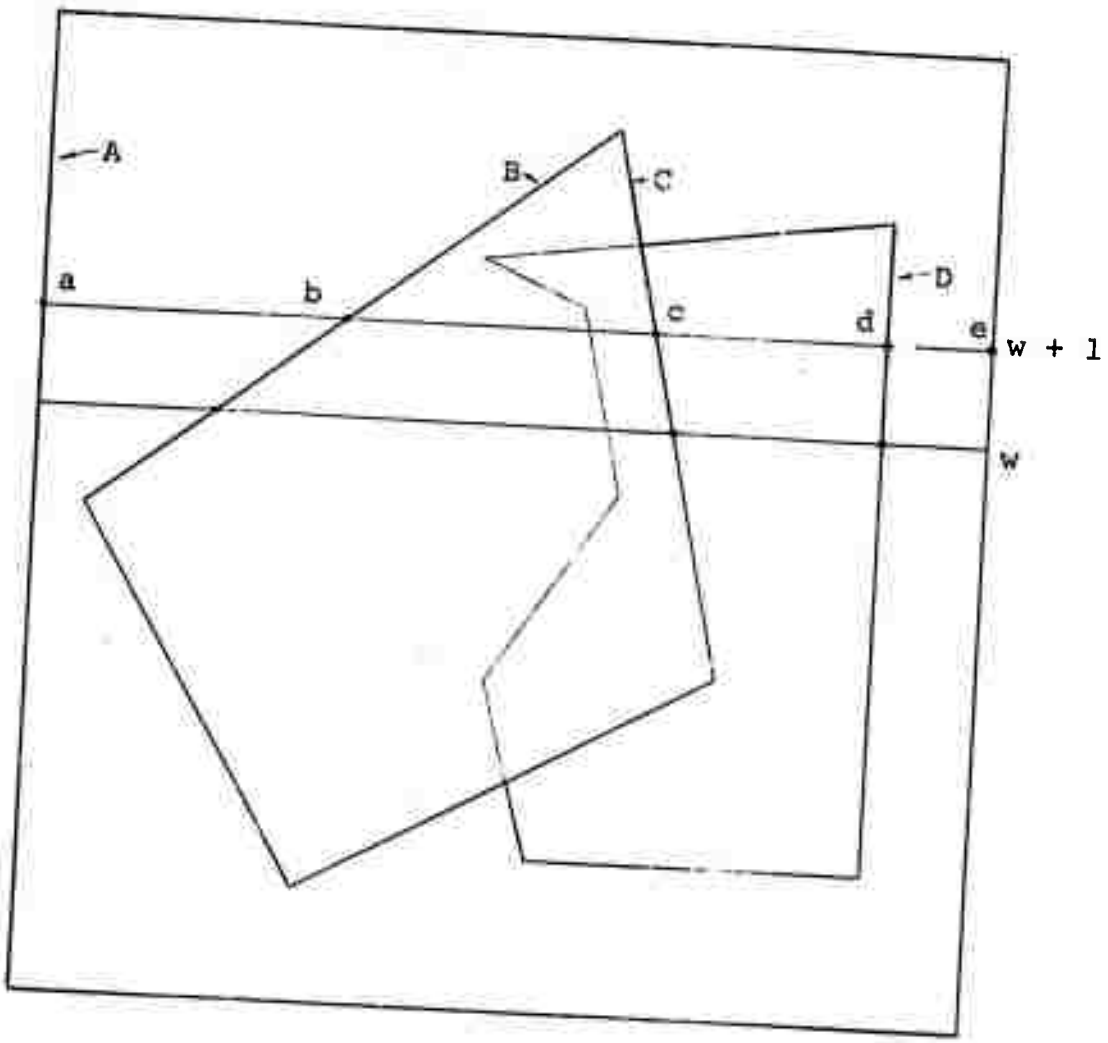


Figure 9
Sample Edges and Sample Points

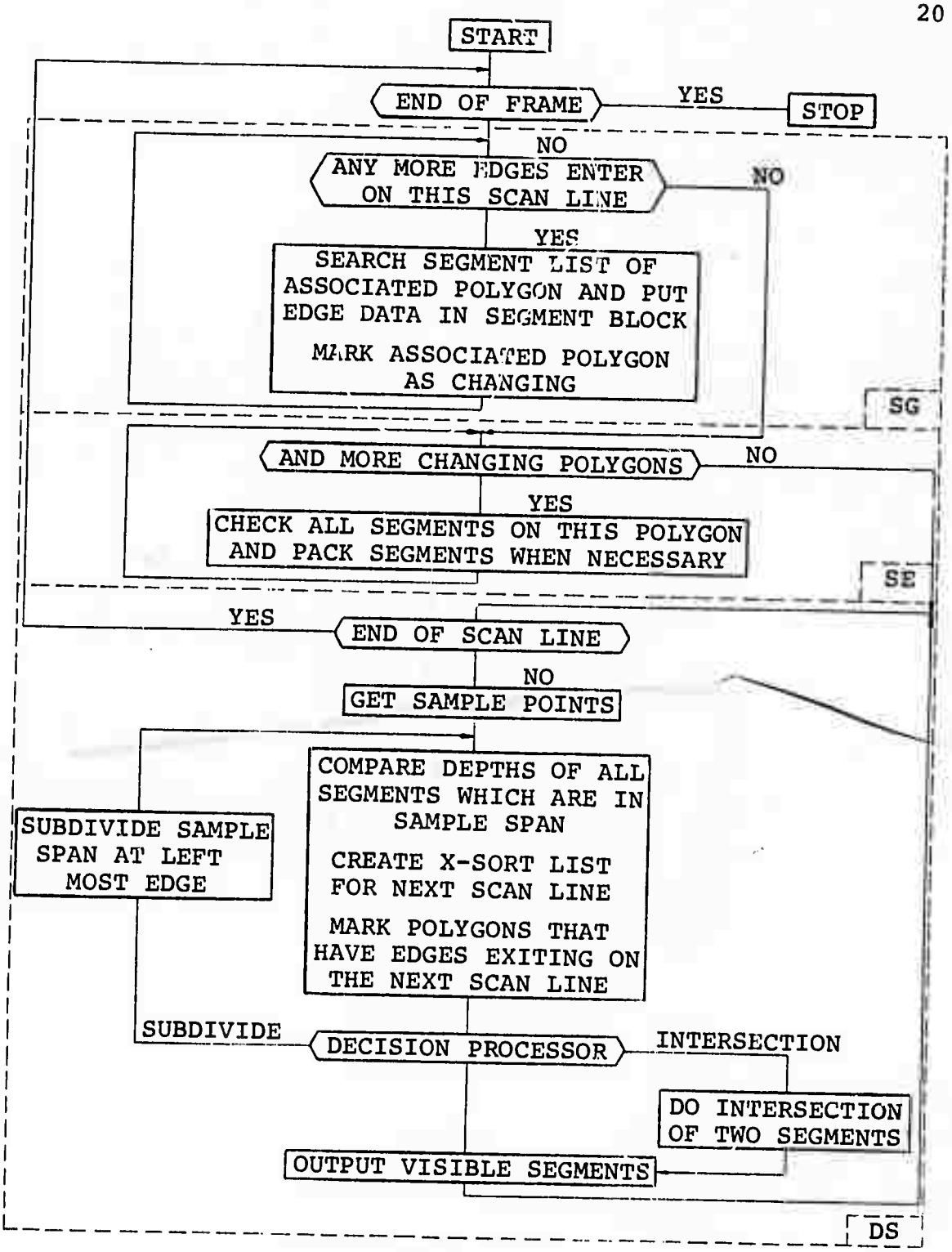


Figure 10

VSG Flowchart

last segment stored in the X-sort list being prepared for the next scan line. If the new segment X value is larger, it is inserted at the end of the list. If it is not larger, the backpointers of the X-sort list are used until the correct location in the list is found. The surprising data is that line-to-line coherence of the ten test objects (Chapter VII) causes 97 to 99 percent of all segments to be inserted at the end of the list. This means that the X-sort list can always remain sorted in X with very little time spent for rearranging segments. (4) Along with the sorting just discussed, the DC must compare the incoming segment against the currently visible segment. If the incoming segment is in front, it will become the currently visible segment. Every time a new sample span is generated, the first incoming segment becomes the currently visible segment.

If the right edge of a segment extends to the right of the right sample point, the segment must be saved for future depth comparisons when the sample span is moved along the scan line. For this purpose the active segments list was created. Segments are put in the list from the X-sort list and remain only as long as the right edge of the segment is to the right of the left sample point. Therefore, in addition to segments read from the X-sort list, the DC also compares depths of segments read from the active list.

G. Segment Clipping

When two segments are being compared, a clipping algorithm is applied to each of the two segments simultaneously. Figure 11 illustrates the procedure. The two lines represent the segment values on the current scan line. As the Z values of a segment decrease, the segment becomes closer to the observer. Two X clipping values must be obtained. X_{lclip} is defined as the right most left edge in the sample span, and X_{rclip} as the left most right edge in the sample span. If a left edge does not lie in the sample span, the left sample span value is taken as X_{lclip}. In Figure 11, the X value of 'e' becomes X_{lclip} and the X value of 'b' becomes X_{rclip}. A set of registers is then chosen for the left and right clip points of both lines and loaded as in Figure 12.

Since Z_{max} and Z_{min} are stored (not Z_{left} and Z_{right}), an additional bit must be kept which is the sign of $(Z_{left}-Z_{right})$. This bit is used to distinguish the relationship of Z_{left} and Z_{right} to Z_{max} and Z_{min}. Figure 13 shows a more complete gating of the registers contained in dotted box #1 of Figure 12. S of Figure 13 is:

$$S=(XA-X_{lclip}+XB-X_{lclip})/2 \qquad (2)$$

or

$$S=(XA+XB)/2-X_{lclip} \qquad (3)$$

But (XA+XB)/2 is the midpoint (XM) of the line ab.

$$S=XM-X_{lclip} \qquad (4)$$

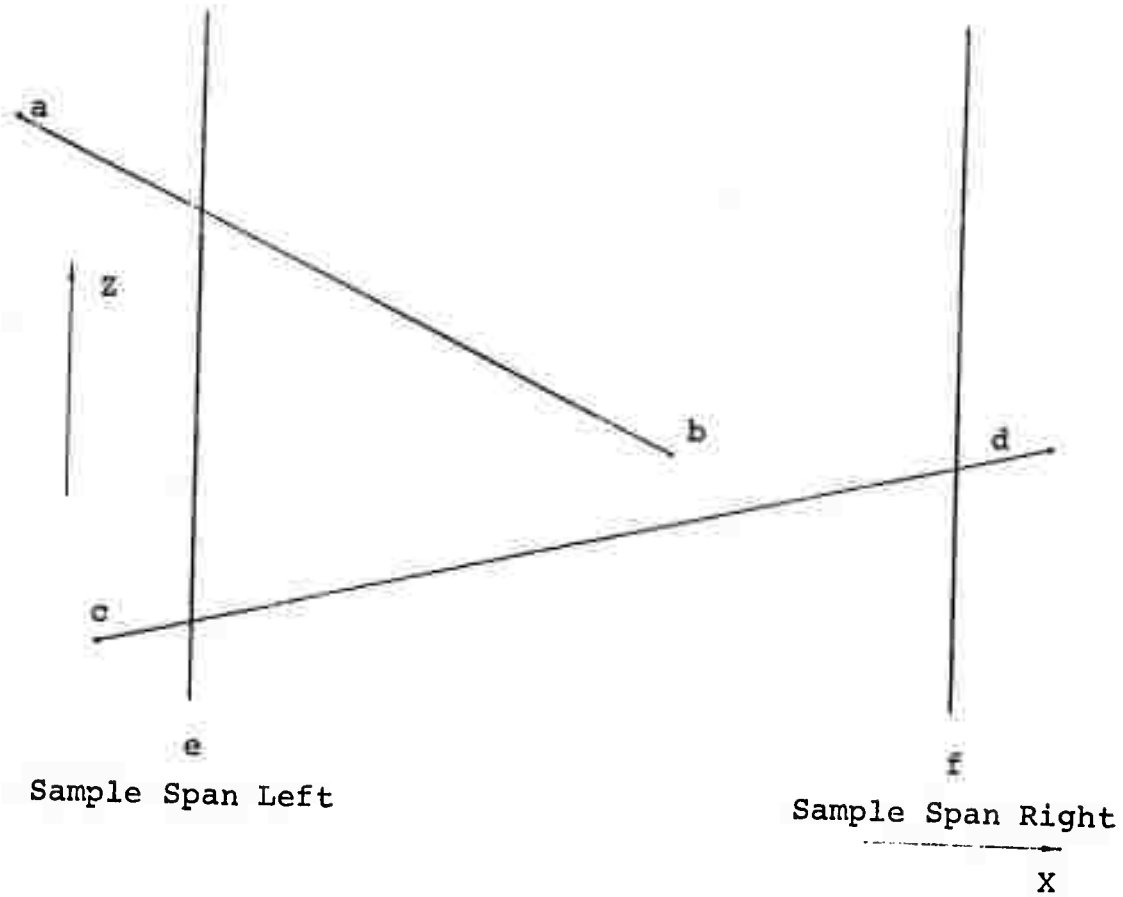


Figure 11
Two Segments on a Scan Line

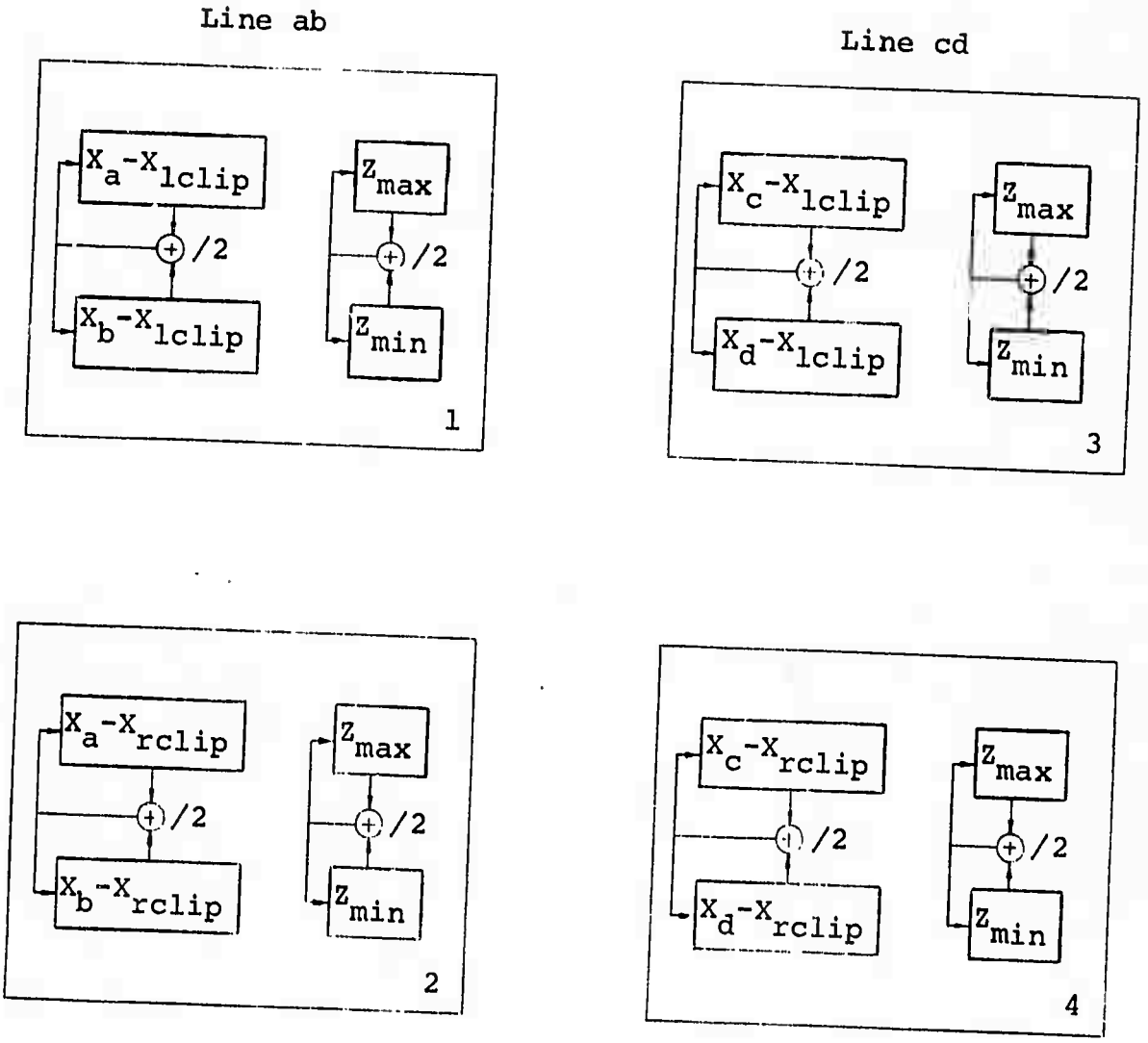
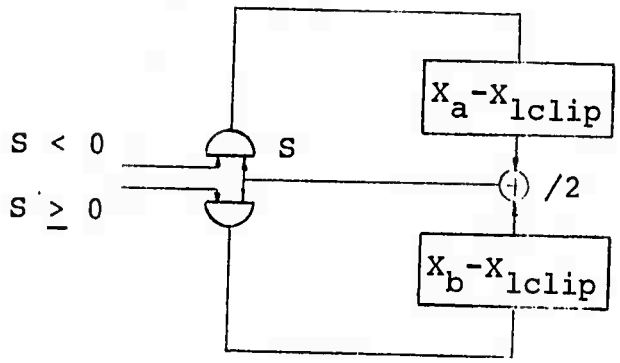


Figure 12
Arithmetic Unit for Depth Comparator



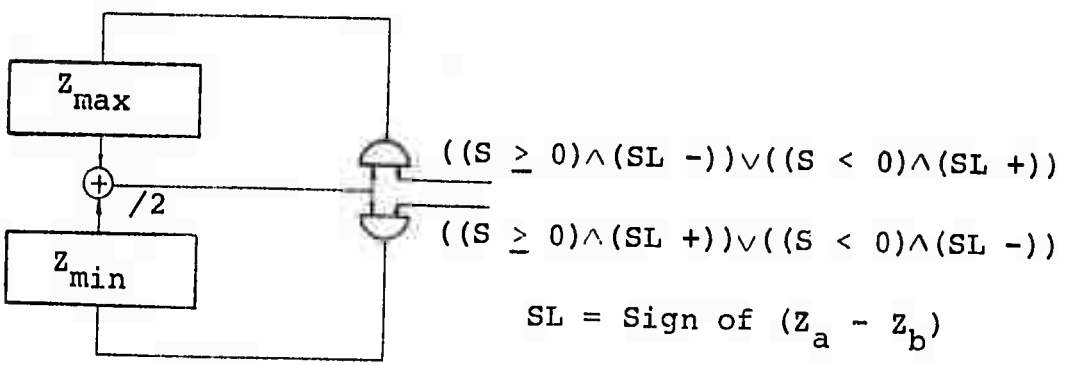
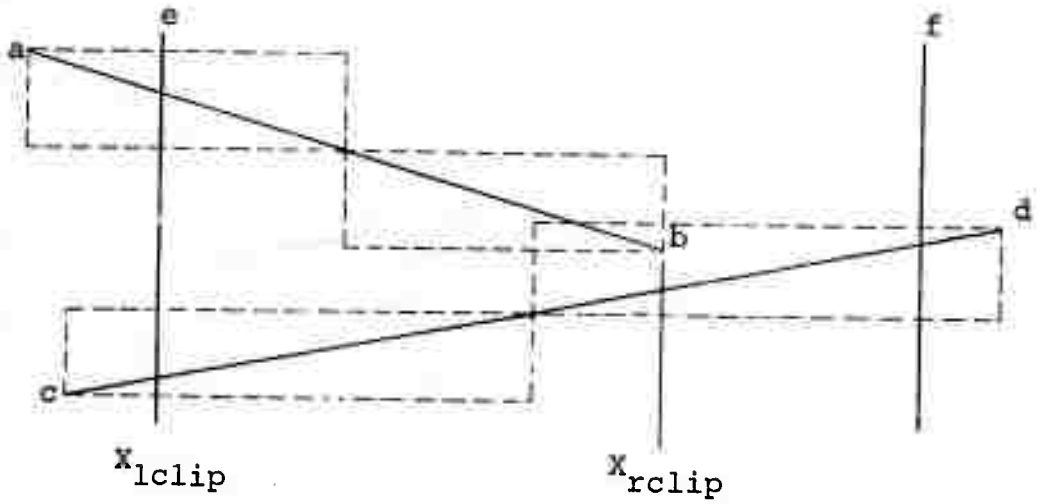


Figure 13
Gating of a Single Quadrant

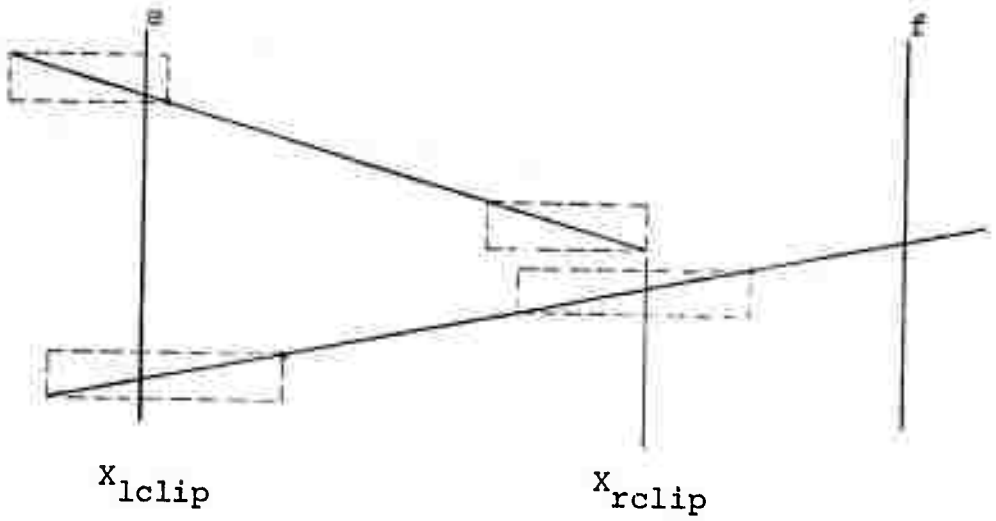
If S is greater or equal to zero, the midpoint is on X_{lclip} or to the right of X_{lclip}. Then the registers containing X and Z of the previous point to the right of X_{lclip} will be replaced with the midpoint of line ab which is closer to X_{lclip}. A similar argument applies if S is less than zero. A more complete description of this clipping process used in a line drawing system is described by Sproull [8].

In Figure 14, succeeding clipping cycles are applied to the two segments of Figure 13. Let Z_{max1} be Z_{max} of quadrant 1 in Figure 12. Z_{min1}, Z_{max2}, Z_{min2}, Z_{max3}, Z_{min3}, Z_{max4} and Z_{min4} are similarly defined. If ($Z_{max1} < Z_{min3}$), line ab is in front of line cd at X_{lclip}. However, as in Figure 14 after one clip cycle, then ($Z_{max3} < Z_{min1}$). Therefore, line cd is in front of line ab at X_{lclip}. Exactly the same argument applies to X_{rclip}, and after two clip cycles line cd is found to be in front of line ab. Since line cd covers line ab everywhere between the sample points 'e' and 'f', it then becomes the currently visible segment.

Many times when lines intersect, or in the case shown in Figure 15, a single currently visible segment cannot be found. In this case a box is made just large enough in X and Z to encompass the two or more lines in question. The amount of data to remember a box description is the same as the amount to remember a line. Also a bit is set declaring a visible box instead of a visible segment. If later a



First Clip Cycle



Second Clip Cycle

Figure 14
Clipping of Segments

segment is found to be in front of the box as in Figure 16, then it becomes the current visible segment and replaces the visible box. The processor continues until all segments that exist in the span are checked. When this is completed, control is passed to the Decision Processor.

H. Decision Processor (DP)

The DP decides whether a visible segment can be put in a display file or if the sample span must be subdivided in some manner and the Depth Comparator started again. If the DP finds there is a visible segment from the DC, it outputs the corresponding segment to the display file. If the DC discovered a visible box, and any of the visible segments in the box have an edge existing within the sample span, the right sample point is set to the X value of that edge (subdivision), and control is passed back to the DP. For instance, the DP would cause the control to subdivide at X=a for segments in Figure 17.

If no edges exist between the left and right sample points, two conditions can exist: (1) For more than two segments existing in the visible box as in Figure 18, the sample span is divided in half. That is, the right sample point is moved half way toward the left sample point. After this subdivision process, control is passed back to the DC again. (2) If only two segments exist in the box, the condition is the intersection case of Figure 19. The

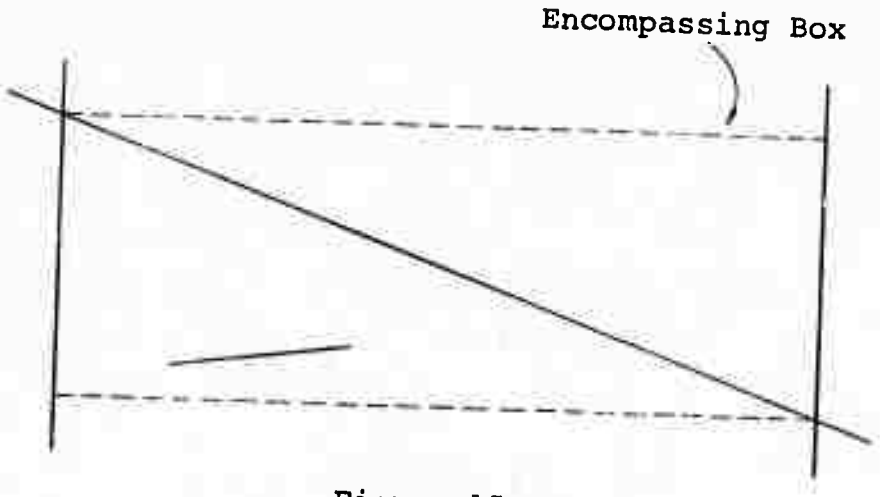


Figure 15
Boxing of Segments

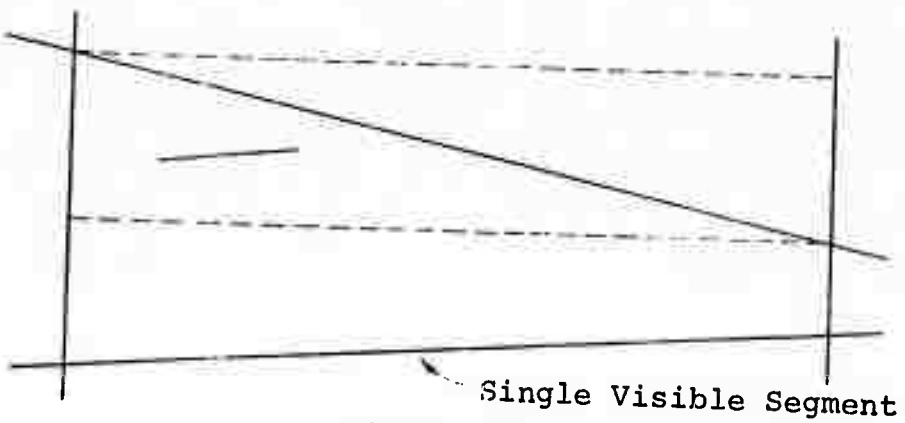


Figure 16
Elimination of Visible Box by Visible Segment

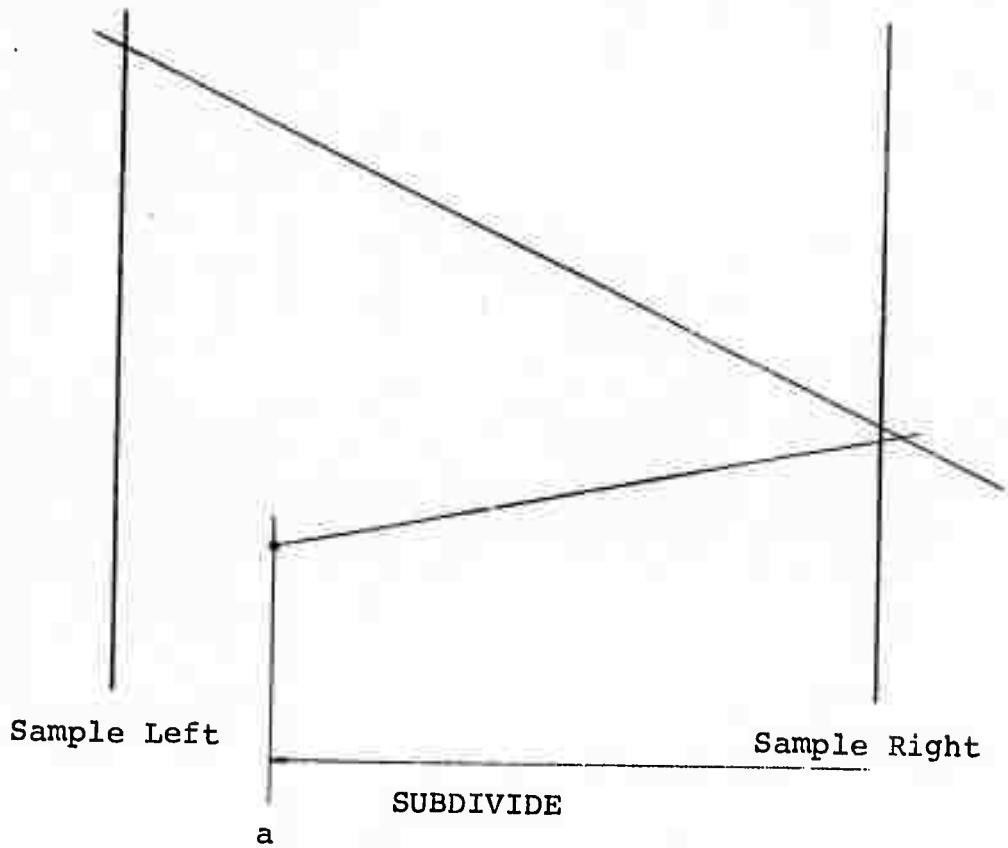


Figure 17
Subdivision

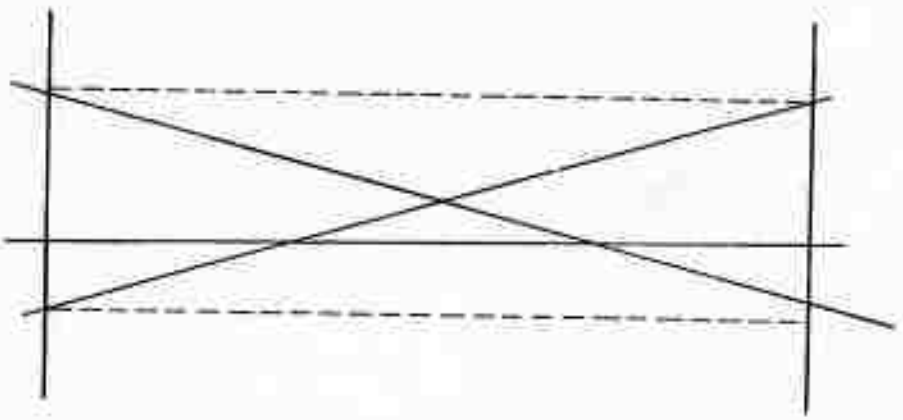


Figure 18
Three Potentially Visible Segments

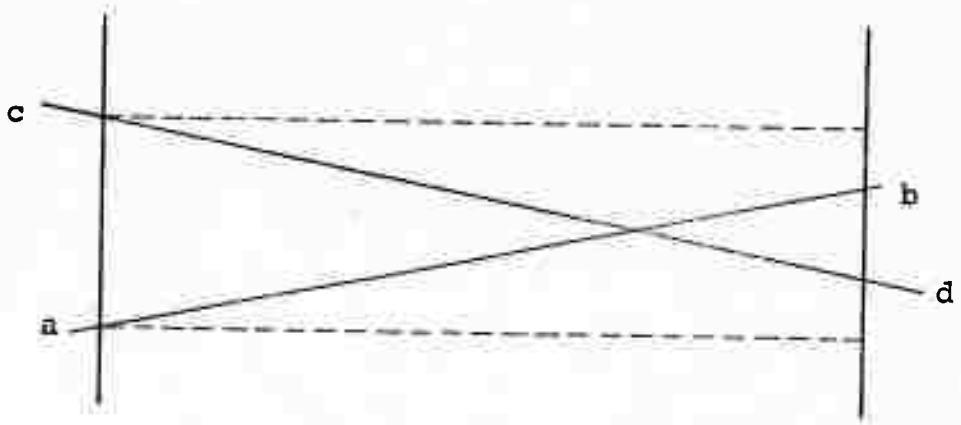


Figure 19
Intersecting Segments

same clipping hardware used for depth comparisons can also be used for calculating the intersection of these two lines.

I. Intersecting Segments

The intersection calculation is done in two stages. First, the registers of Figure 12 are loaded exactly in the same manner as for the DC. However, instead of terminating when the Z_{max} and Z_{min} tests are satisfied, the adders run until all registers contain either 0 or -1. When the registers reach this state, Z_{max1} will hold the Z value of line ab at X_{lclip}, Z_{max2} the Z value of line ab at X_{rclip}, Z_{max3} the Z value of line cd at X_{lclip}, and Z_{max4} the Z value of line cd at X_{rclip}. Figure 19 has been reduced to the problem represented in Figure 20.

For the second stage, the problem can be solved by loading the registers in the manner shown in Figure 21. Because of the intersection, Z_1 and Z_2 will have opposite signs. Therefore, after each add cycle the Z sum is stored into the Z register which has the same sign as the sum. The X registers will also be stored in the same direction determined by the Z sum. After $[\log_2(X_{rclip}-X_{lclip})]$ add times, X_1 and X_2 will both contain the X value of the intersect of the two segments.

A block from free storage is obtained at this point and the X intersect value and the pointers to the two segments causing the intersection are stored as data in an implied edge list. When the program proceeds to the next scan line,

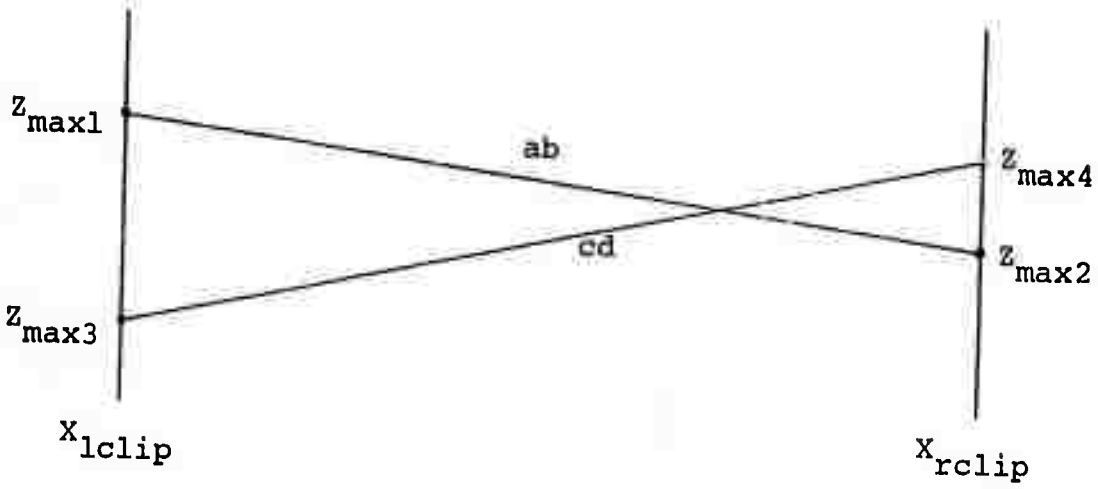


Figure 20
Intersecting Segments Clipped to X_{lclip} and X_{rclip}

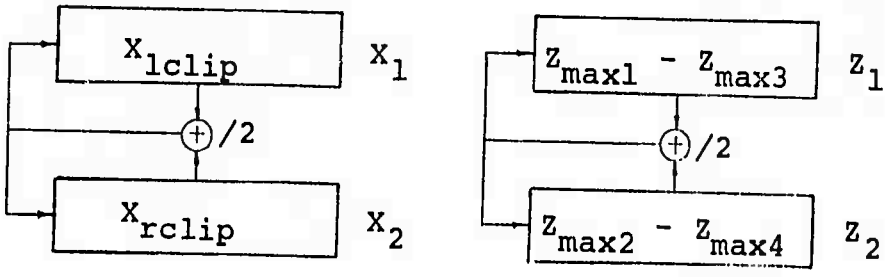


Figure 21
Registers for Finding Intersection

the intersect will again be calculated. The difference between the intersect on this scan line and the intersect calculated on the previous scan line can be used as the increment of the implied edge. This edge can now be treated as any other visible edge and used for determining sample points. If, on a scan line, an implied edge is found to be no longer visible, the block is returned to free storage.

J. Building the Sample List

The DP has one other task. That is, to tag the visible edges (determined in the DP), and put them in the sample list. Upon completion of the DP, control is either passed to the SSG if subdivision did not occur, or to the DC if subdivision did occur.

CHAPTER IV

FRAME-TO-FRAME COHERENCE

This algorithm can easily take advantage of frame-to-frame coherence of pictures. For instance, in a movie if an edge is visible in one frame, it will usually be visible in the next frame. If an edge is found to be visible on the scan line it enters on, the edge block (see Figure 4) is tagged as visible. This means one additional bit must be stored in each edge block. Also two pointers to each edge block must be stored in the segment blocks. Then when the next frame is being processed and an edge was found to be previously visible, the initial X value of the edge is then used as a sample point. The frame-to-frame coherence algorithm was used on some of the earlier versions of the program. However, the scan line-to-scan line coherence was so efficient that the frame-to-frame coherence only decreased the number of memory references by about 0.1 percent. Because of this, it was not implemented in later programs.

CHAPTER V

RELATIONSHIP WITH OTHER ALGORITHMS

On the basis of generality of object descriptions, this new algorithm is as good as or better than the others mentioned in the introduction. Convex or non-convex polygons of any number of sides can be used. The algorithm allows polygons to penetrate one another without any pre-processing checks.

Since planar equations are never used for depth sorting, the algorithm can not tell if the points of the polygons lie on a plane. It always assumes a linear interpolation between the edges on a scan line. However, when shading a polygon a discontinuity in shading can be created. For example, if the vertex between scan lines 'a' and 'b' of Figure 6 were not on the plane described by the other three vertices, the linear depth calculations between edges would show a discontinuity in the shading between the two scan lines. Furthermore, the line of discontinuity would always remain horizontal even if the polygon were rotated. Also, since segments are only checked when edges enter or exit, edges of a single polygon should never cross each other. If they do cross, however, a local error will occur in the picture only where that polygon exists and if that polygon is visible. Consequently, points of a polygon

should lie on a plane. (Points not on a plane can introduce edges that cross).

Like Warnock's algorithm, this new algorithm is also non-deterministic, but on a scan line level. For instance, a sample span on a scan line is assumed to have one covering polygon. If it does not, the sample span is made smaller until finally a span is found which is covered by a single polygon.

Romney used an ordering scheme for taking advantage of scan line-to-scan line coherence. He did not allow intersecting triangles. Therefore, as long as the intersection of the edges of triangles on the current scan line were in the same order as on the previous scan line, the same triangles that were visible previously would be visible on this scan line. However, as soon as the order changed, the remainder of the scan line had to be depth sorted. The coherence ordering made a great difference in the speed of his algorithm.

If intersections are allowed, as in the new algorithm, this ordering of edges no longer holds for determining visibility. Therefore, the sampling process described in Chapter III-D was developed. It has the further advantage that even when the order changes, the previously calculated sample points for the remainder of the scan line are still valid.

CHAPTER VI

DEVELOPMENT OF THE NEW ALGORITHM

As is usually the case in the development of new algorithms, the process was evolutionary. Successive algorithms were developed, tested, and improved upon. The history of this algorithm can be divided into six distinct steps. These programs are called VSG1, VSG2, etc.

1. The first step used edges on each scan line. The edges were sorted in X separately, and after sorting they were read in order. Every time an even number of edges was found associated with a polygon, a segment block was created from free storage. Finally, the segments were depth sorted for visibility.

2. VSG2 linked the edges together with pointers after sorting in X. This eliminated the creation of segment blocks on each scan line.

3. VSG3 took the edge data and created segment blocks only when edges entered on a scan line. These segments are described in Chapter III-F. Since there are one half as many segments as edges, the X-sort on each scan line is twice as fast as in VSG2. Also, edges no longer needed to be linked together on every scan line.

4. VSG4 eliminated the X-sort which was done separately before the depth sorting. The X-sort and depth sort were done simultaneously on each scan line.

5. The four previous algorithms used planar equations and a multiplier for calculating depths of the polygons. A divider was also required for finding the intersect of two polygons. VSG5 replaced the arithmetic unit with the midpoint clipping simulation described in Chapter III-E.

6. Up to this point all algorithms used a bucket sort as described by Romney [5] for sorting segments in X. This final algorithm used the assumption that a sorted list will remain sorted by interchanging only a few segments when proceeding from one scan line to the next.

CHAPTER VII

TEST DATA

Ten objects were chosen to represent various complexities of pictures. Figures 22-31 contain pictures of the objects. Each object has two pictures. One shows all edges in the picture and the other shows the objects after visible surfaces are found and shaded.

A. Objects

Object 1, Penetration: The object is relatively simple but has many intersecting planes.

Object 2, E-S: Many edges abound in the picture and a great amount of visible complexity exists.

Object 3, Low Area: Although intersections abound, the picture only occupies a small area.

Object 4, Cube1: Twenty-five cubes exist, but only the front cube is visible.

Object 5, Cube2: Object 4 has been rotated so that parts of all twenty-five cubes are visible. An enormous amount of visible complexity exists.

Object 6, Shape1: This object is made up of many long and narrow polygons which are long in the X direction.

Object 7, Shape2: Object 6 has been rotated so the polygons are long in the Y direction. These two objects are to show what effect the object orientation can have

on the scanning process.

Object 8, Sheet: This is a wavy object made up of triangles. Everything is at least partly visible.

Object 9, Simple1: This object is made up of a large cube encompassing a sphere and intersecting cubes.

Object 10, Simple2: Object 9 has been changed slightly so the sphere intersects the cube and is partly visible.

B. Statistics

For each of the VSG algorithms mentioned in Chapter VI, statistics were gathered. These statistics included data about the object (number of polygons, etc.), computation required, memory reference counters, and various other counters. Appendix II contains a list of statistics. At the beginning of each set of statistics for a particular algorithm, there is a table describing the various counters. Figure 32 contains a table of statistics that have been extracted for the Penetration object (Figure 22). The statistics of the six various changes in the algorithm are shown for that object. The table in Figure 33 shows a cross section of statistics for all the objects with the final algorithm.

C. Analysis

Before any statistics were gathered, arithmetic computation was suspected to be the bottle-neck in solving the hidden line problem. Statistics, however, showed that

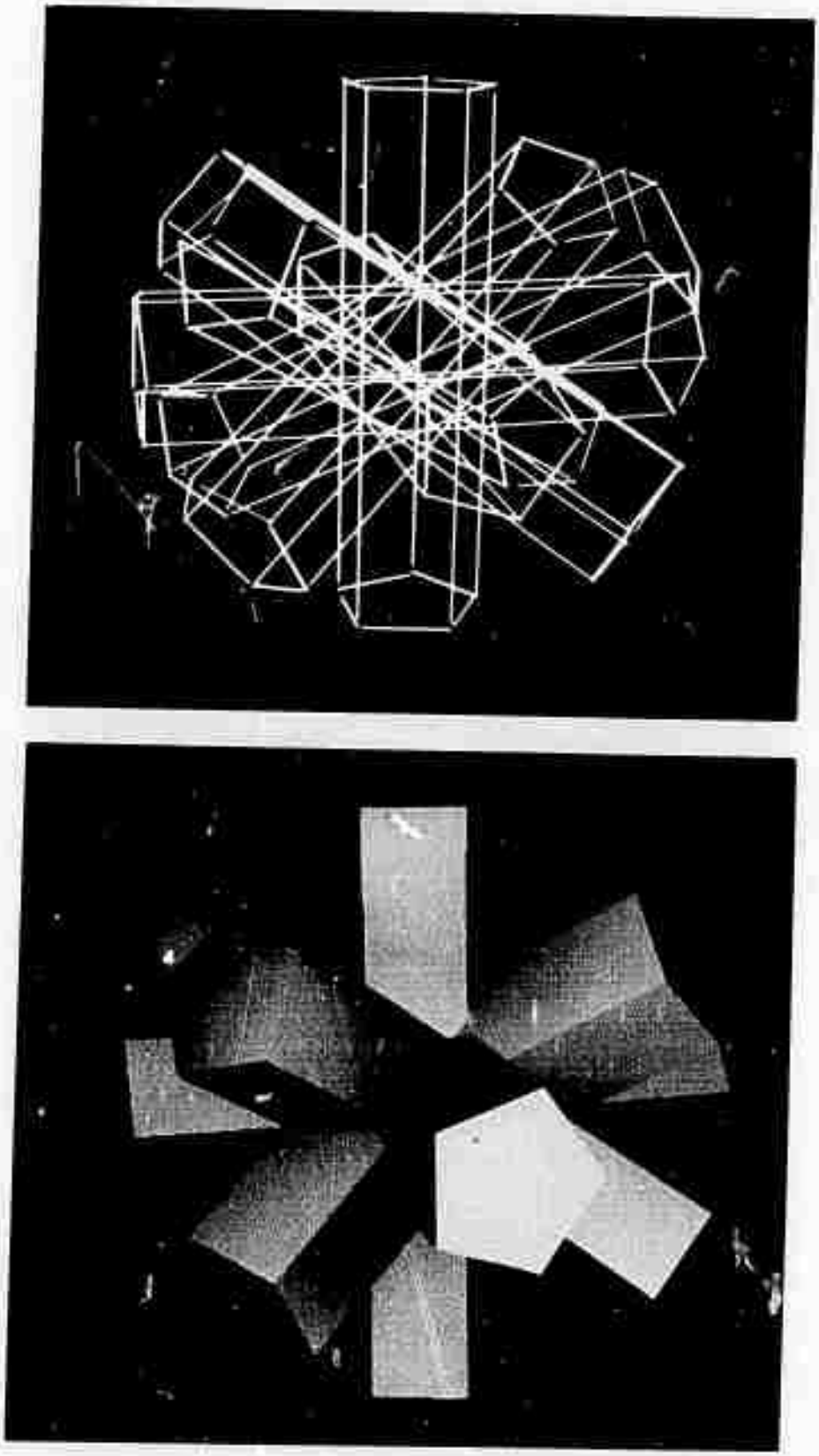


Figure 22
Object 1: Penetration

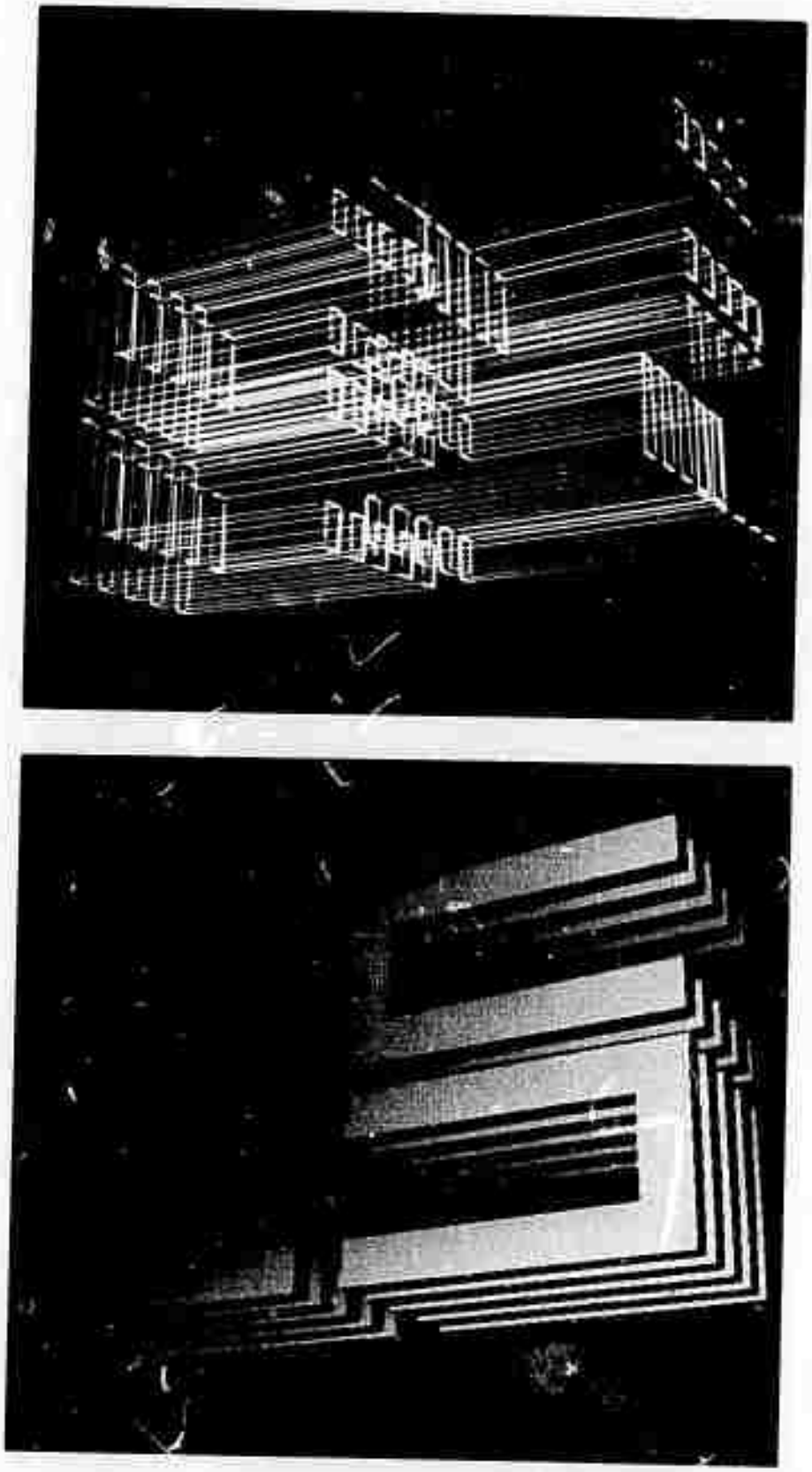


Figure 23
Object 2: E-S

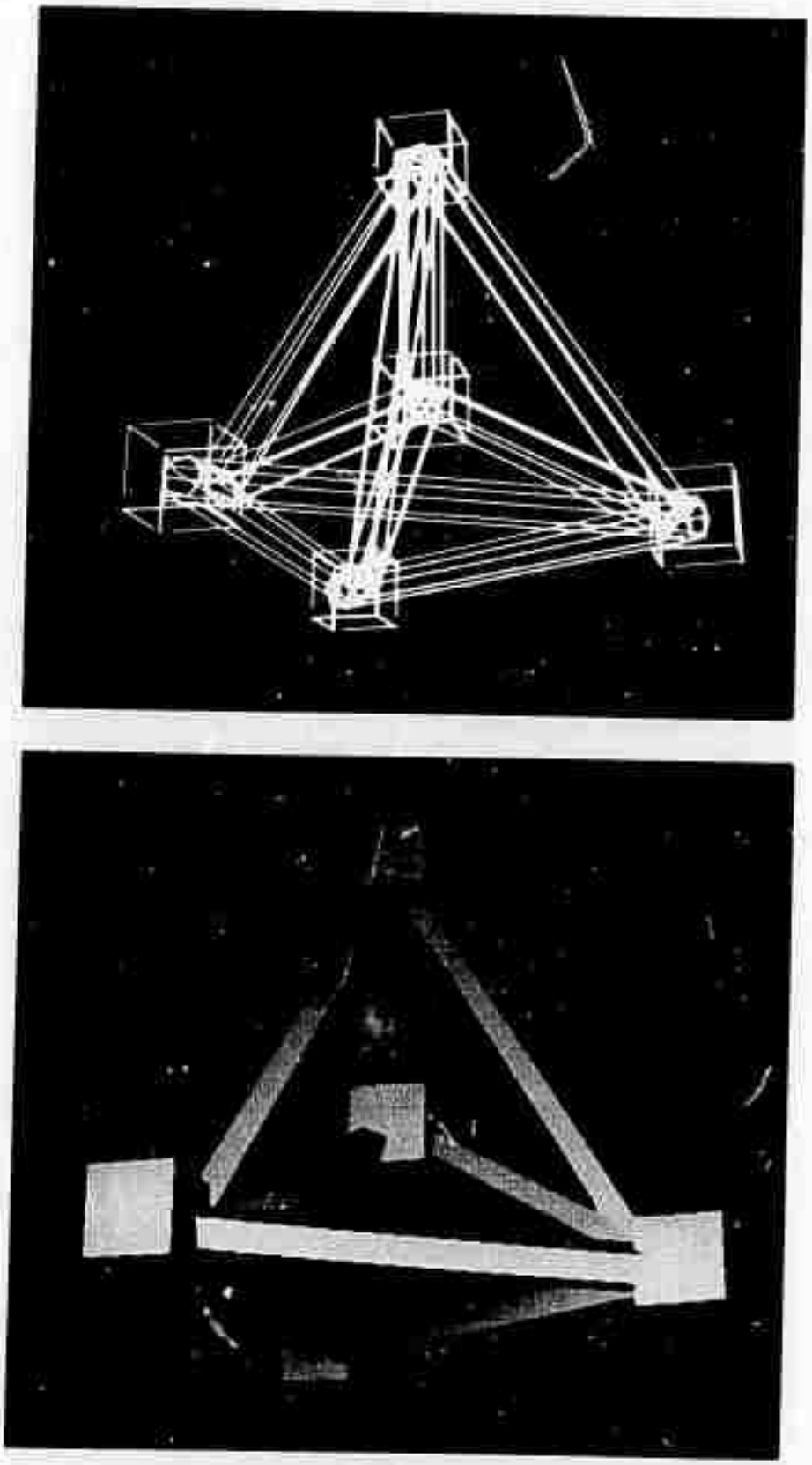


Figure 24
Object 3: Low Area

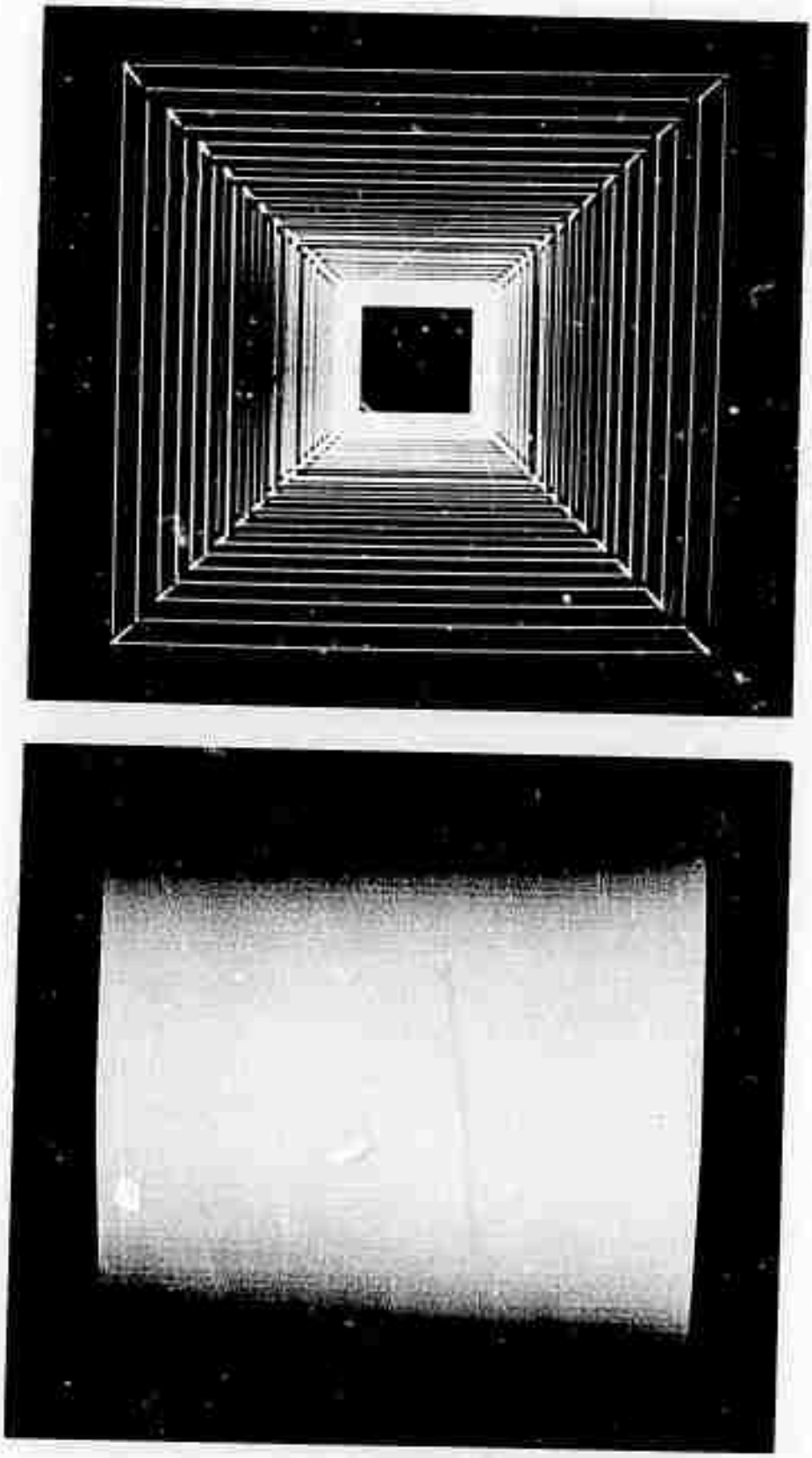


Figure 25
Object 4: Cube1

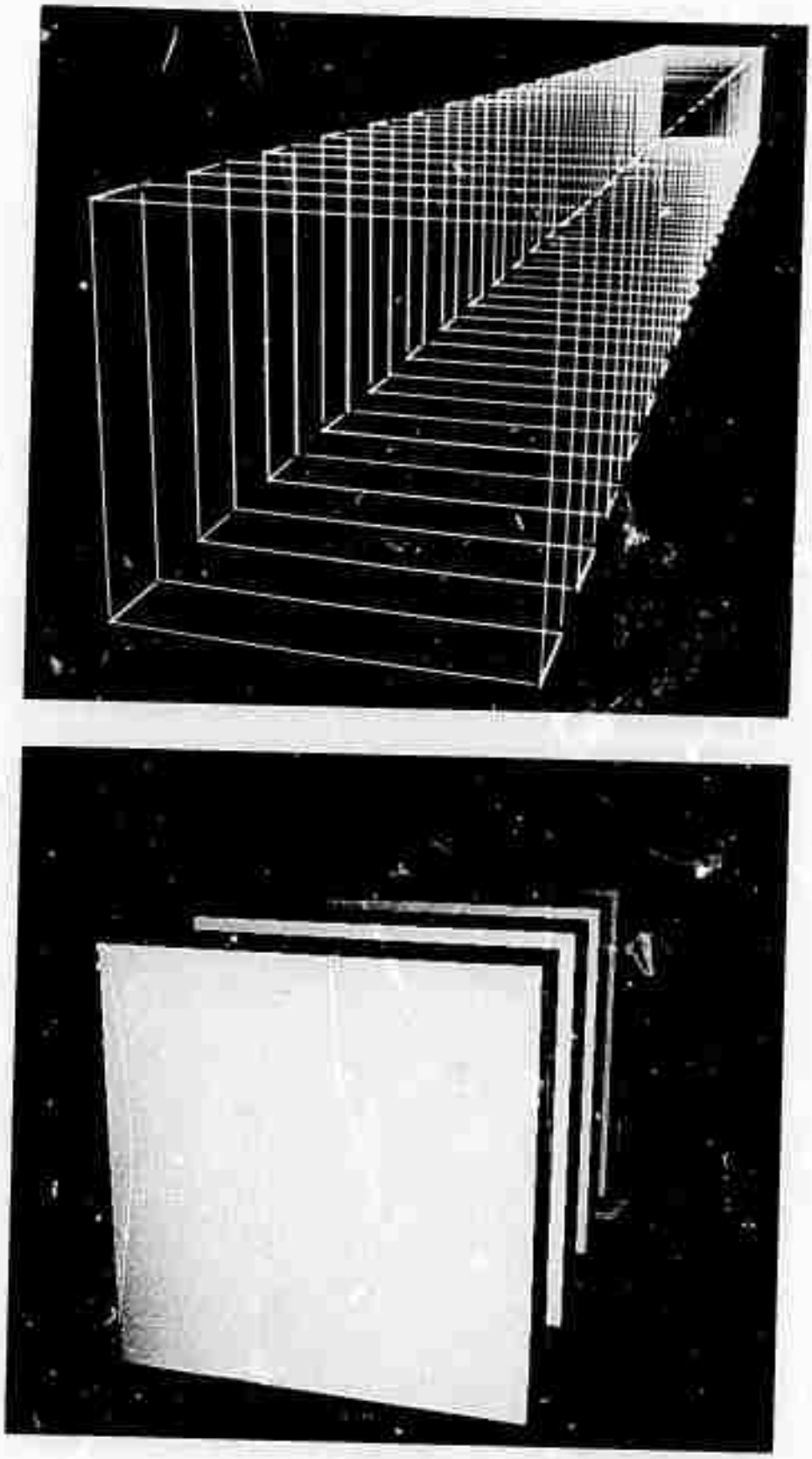


Figure 26
Object 5: Cube2

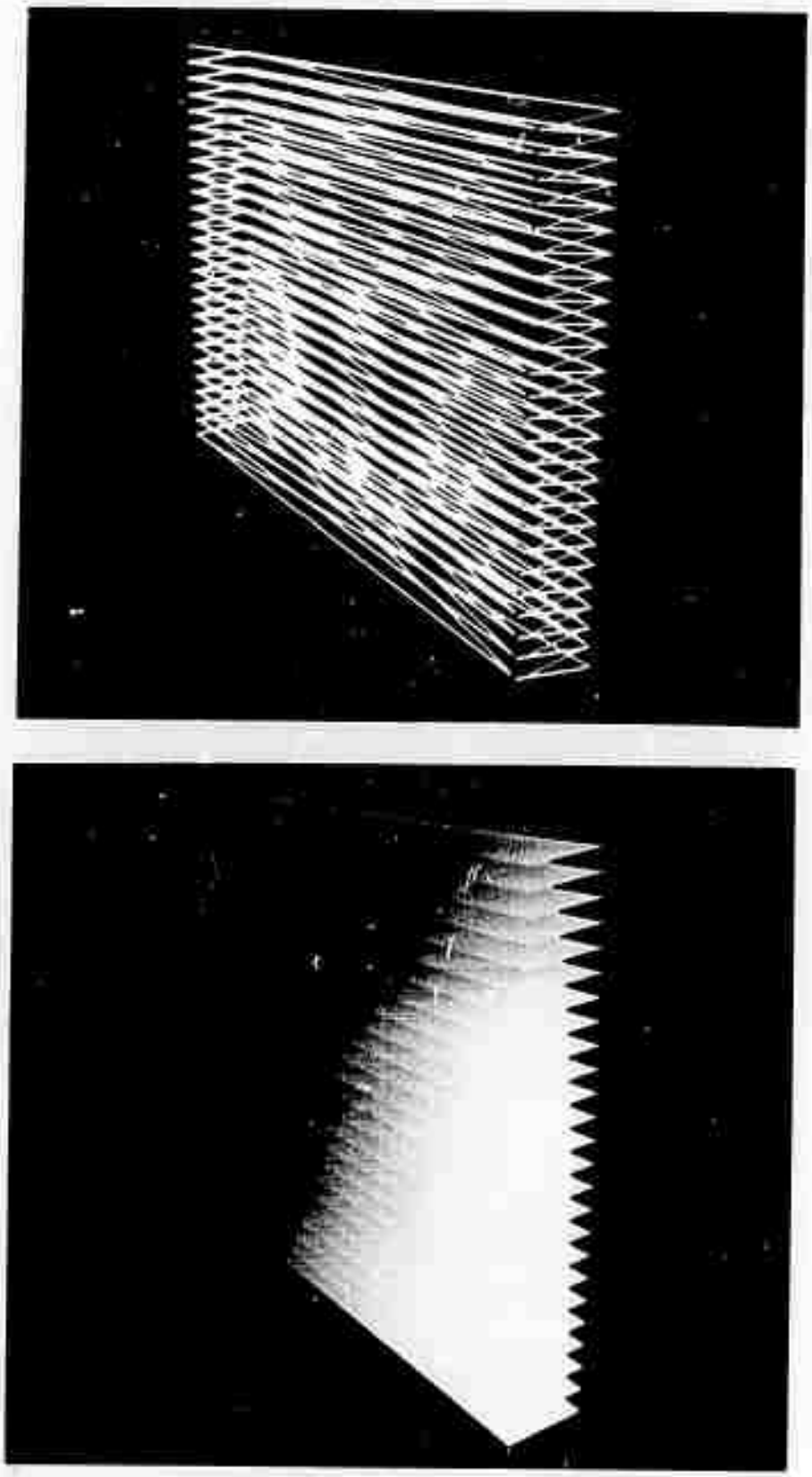


Figure 27
Object 6: Shape1

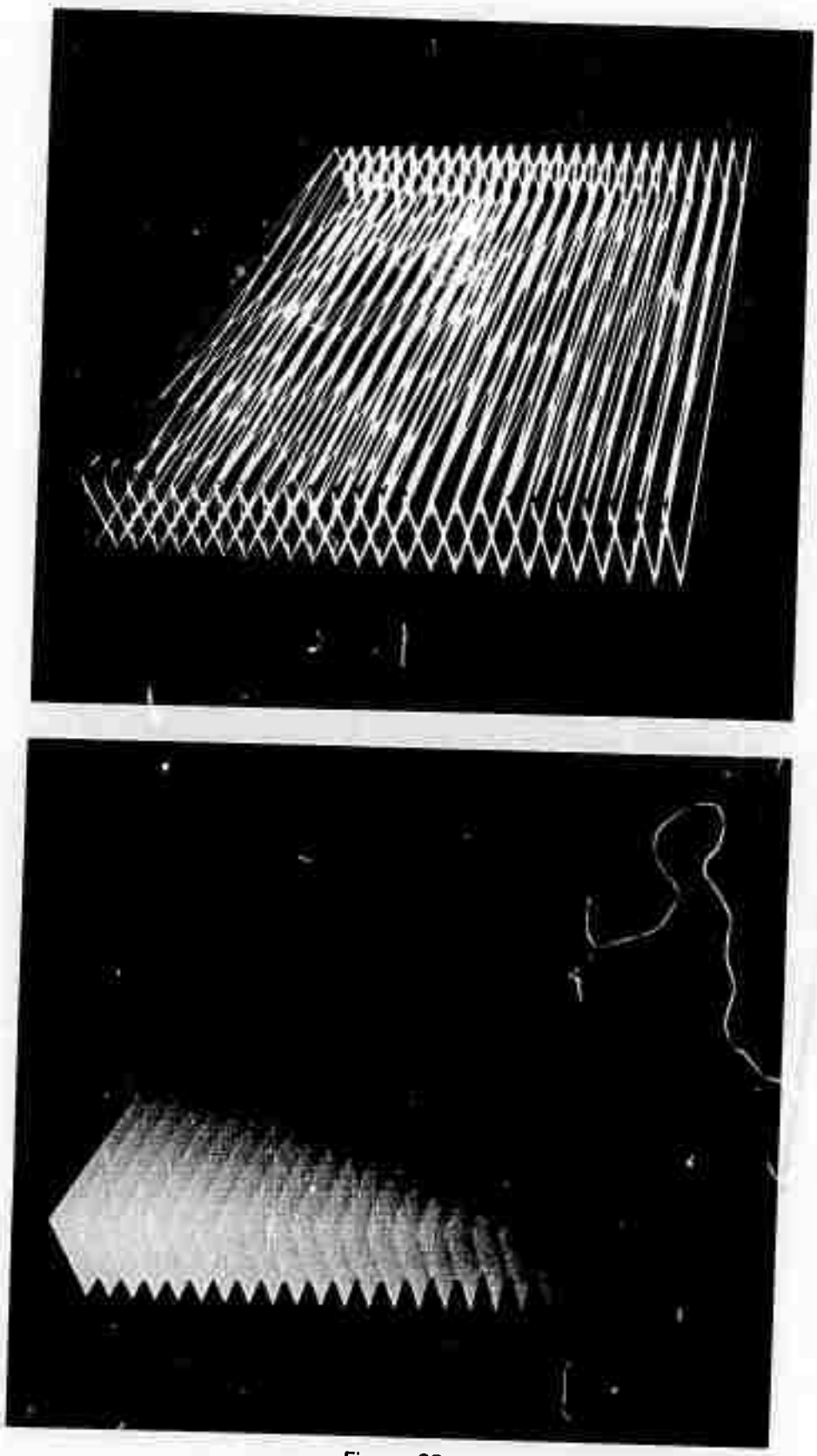


Figure 28
Object 7: Shape2

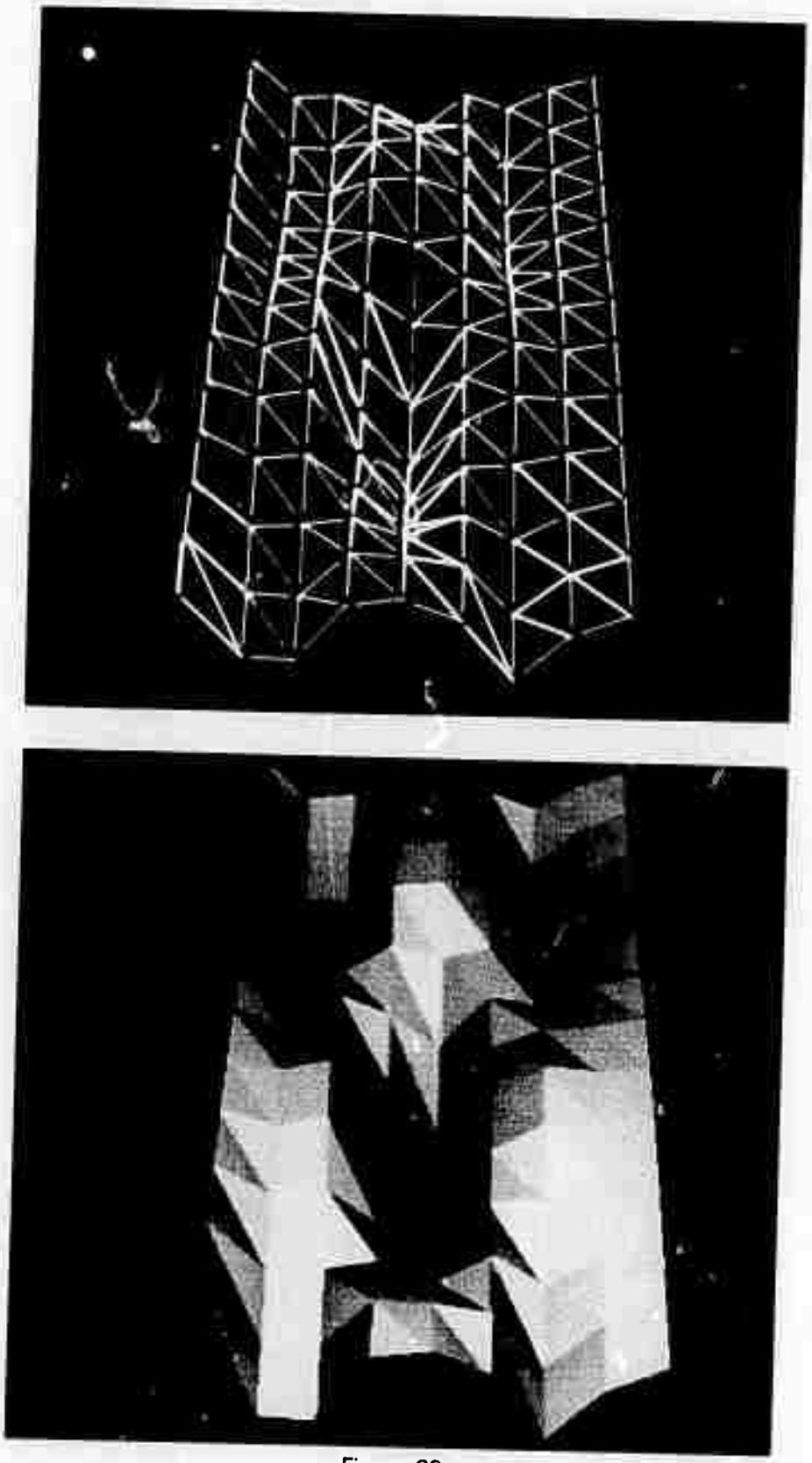


Figure 29
Object 8: Sheet

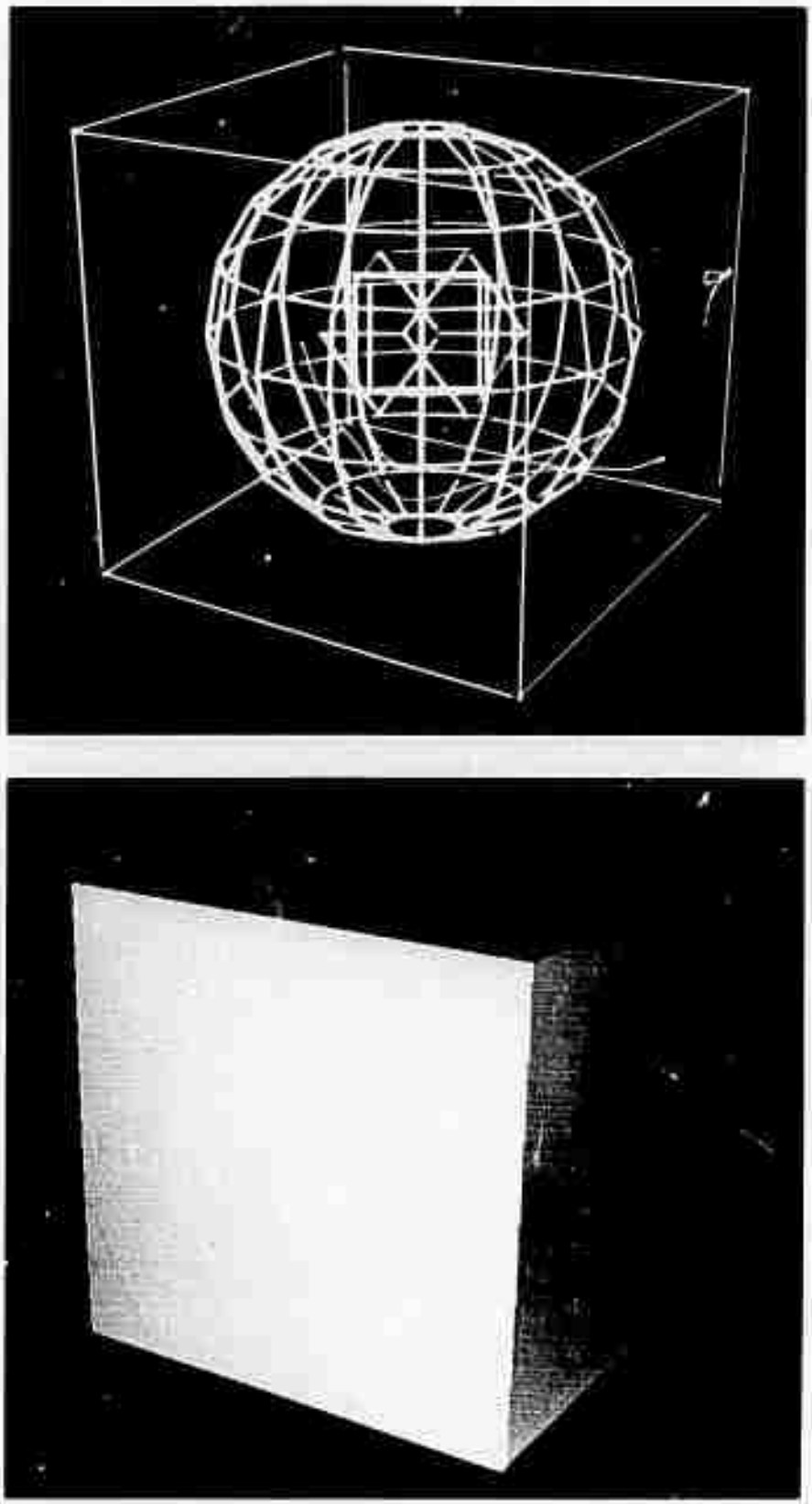


Figure 30
Object 9: Simple1

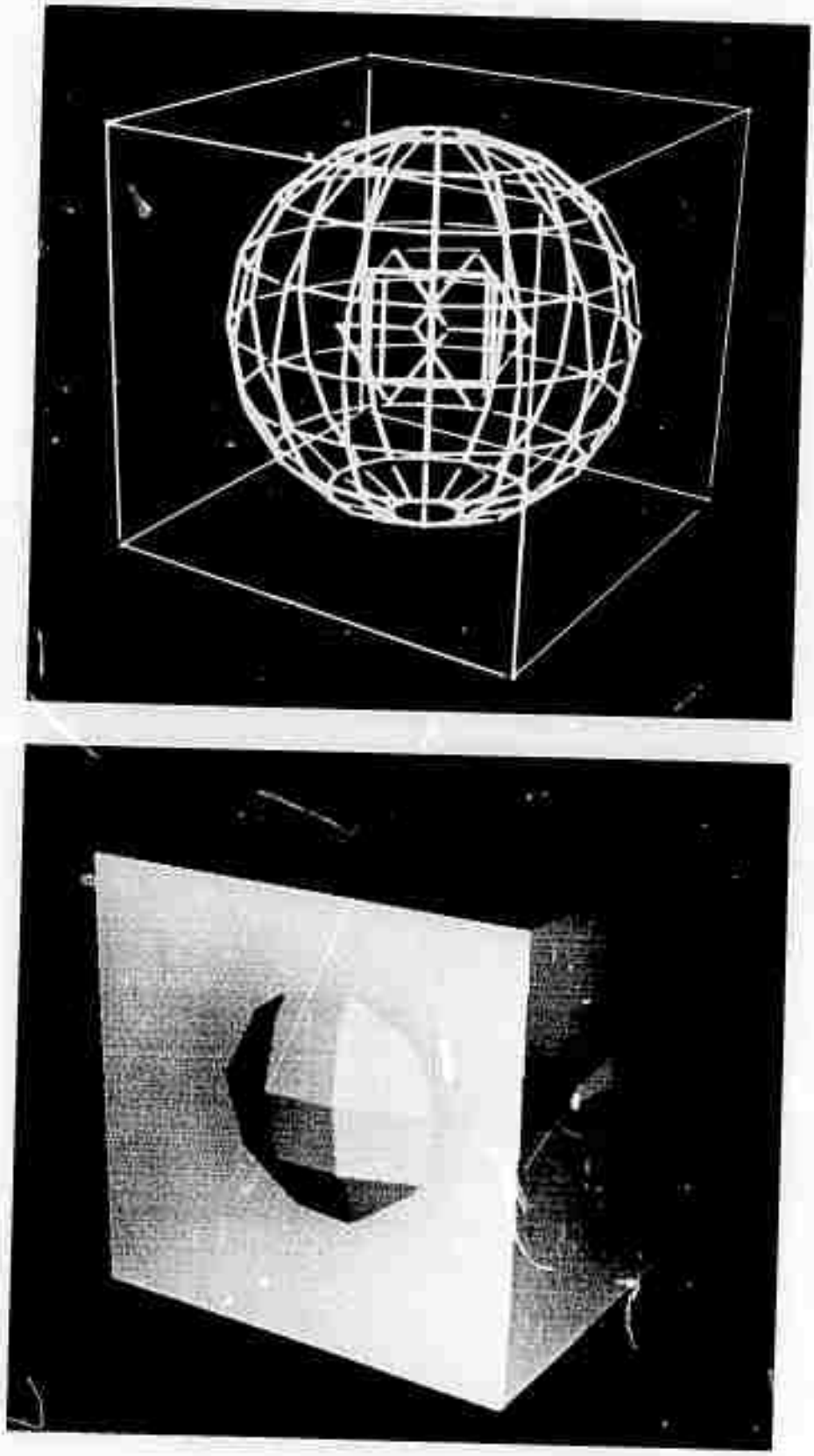


Figure 31
Object 10: Simple2

	VSG1	VSG2	VSG3	VSG4	VSG5	VSG6
1	137607	101892	37919	21730	21224	24291
2	9172	112	38	54	54	56
3	76	44	25	24	23	23
4	7068	6975	6839	9210	-	-
5	26	22	22	15	-	-
6	-	-	-	-	35604	32925

1. Number of memory references required
2. Number of total memory blocks used
3. Maximum number of blocks used at a time
4. Number of multiplications for depth test required (If multiplier-divider used)
5. Number of divisions for intersections required (If multiplier-divider used)
6. Number of addition cycles required for depth comparisons (If multiplier-divider not used)

Figure 32

Statistics of the Penetration Object

for the Six Algorithms

	PENETRATION	E-S	LOW AREA	CUBE1	CUBE2	SHAPE1	SHAPE2	SHEET	SIMPLE1	SIMPLE2
1	49	200	100	150	150	100	100	192	148	148
2	105	480	210	300	300	201	201	308	308	308
3	8802	15278	9154	7158	15410	12424	25590	12500	6172	6664
4	14	0	31	0	0	0	0	0	0	21
5	3844	7052	4193	1238	8221	5630	12023	6935	1652	2383
6	56	246	109	25	121	50	75	250	66	90
7	23	40	31	25	46	24	50	31	18	22
8	24291	47030	24071	7608	80945	34314	79762	26970	9039	14926
9	2.75	3.07	2.62	1.06	5.25	2.76	3.11	2.15	1.46	2.23
10	32925	47174	25641	10374	109190	30227	85941	14176	12326	24983
11	3.38	2.59	3.23	2.89	2.05	2.12	2.66	2.07	3.74	4.94
12	2.96%	3.48%	4.06%	0.00%	4.26%	1.69%	0.52%	1.17%	1.12%	1.07%
13	10	30	10	3	1700	10	1600	25	5	7

Figure 33

Statistics of VSG6 for the Ten Test Objects

1. Number of polygon blocks used to describe the object
2. Number of edge blocks used to describe the object
3. Total number of times that edges cross scan lines
4. Number of implied lines
5. Number of output segments
6. Total number of memory blocks required for storing segment information (300 bits/block)
7. Maximum number of memory blocks ever used at one time for storing segments
8. Total number of references to segment blocks
9. Ratio of line 8 over line 3
10. Total number of add cycles required (Includes one add cycle per depth test for loading clipping registers)
11. Average number of add cycles per depth test (Includes one add cycle for loading clipping registers)
12. Percent of time when segments are not put on the end of X-sort list for the following scan line
13. Minimum output segment buffer required for real time display

Figure 33 Continued

memory bandwidth was the critical factor, with the polygon segment block being the most accessed array! For the hardware processor, a special purpose memory would be used where 300 bits could be accessed at one time. With semiconductor memories it is becoming economical to do this.

From the first five different changes in the algorithm in Figure 32, one can see a steady decrease in the number of memory references. The final algorithm, however, produced an increase in memory references due to the X-sort technique described in Chapter III-F. In spite of this apparent increase in memory references for VSG6, the overall number of memory references in VSG5 would have been greater if accesses to the bucket X-sort memory had been counted. The design and cost for such a bucket X-sort memory also were compelling factors in deleting it even though accesses to the segment memory increased.

When the program proceeds from one scan line to the next, each segment block needs to be accessed for incrementing the X and Z values. At this same time, another X-sort list is being sorted in preparation for the following scan line. Segments are read from the beginning of the X-sort list for the current scan line and are usually inserted at the end of the X-sort list which is being prepared for the next scan line. Figure 33 (line 12) shows the percentage of times that segments cannot be inserted at the end of the list, and when the previous segment pointers

must be used for finding the correct position in the list for inserting the segment block. The percentage varies between 0 to 4 percent for the ten test objects. Thus, the overhead of tracing back through a list to keep it sorted is extremely low. Also, no large, expensive, or possibly time-consuming special sorting hardware needs to be used.

Visual complexity is much more important in determining the speed of the algorithm than is the total object description. Object 4 and Object 5 are both sets of twenty-five cubes. However, Object 5 requires over ten times the number of memory references as Object 4. A picture visually identical to Object 4, but containing only one cube, was compared with Object 4. Even though Object 4 contained twenty-five cubes, it only had six times the memory references as the single cube object.

One way of measuring the performance of the algorithm is to create a relationship between the object description and the number of memory references to the segment array. Two memory references (a read from memory followed by a write to memory) are always required to increment the X and Z values of a segment when proceeding from one scan line to the next scan line. From the total number of times edges cross scan lines (line 3 of Figure 33), the minimum number of memory references needed can be calculated from Equation 5.

$$M_{min}=(S*N)/E \qquad (5)$$

where M_{min} is the minimum number of memory references that can be expected. S is the number of memory references required to increment a segment (2). N is the total number of times that edges cross scan lines. E is the number of edges contained in a segment (2). Equation 5 reduces to Equation 6.

$$M_{min}=N \tag{6}$$

Equation 7 is the ratio (R) of M_{total} (the total number of memory references actually used) to M_{min}.

$$R=M_{total}/M_{min} \tag{7}$$

Line 9 of Figure 33 lists the different values of R for the ten test objects.

The clipping of segments for depth sorting is very fast. Line 11 of Figure 33 contains the average number of add cycles required by the clipping registers to satisfy the depth comparison test between two polygon segments (see Chapter III-G). One of the add cycles is for loading the clipping registers. Even counting this, the average number is between two to three add cycles per depth test!

The ten test objects were also used by Stephen McCallister [9] for gathering statistics on different versions of Warnock's algorithm. Comparisons are shown for a particular version which divides an area into four sub-areas using a vertex closest to the center of the large area for the common corner of the four sub-areas. If an area is completely covered by a polygon, is void of all

polygons, or has only one visible edge in the area, it is simple enough to be displayed without further reduction.

Statistics for Object 2, E-S (Figure 23), were gathered. A large data structure was used requiring polygon lists, edge lists, and a vertex and planar equation array. Each polygon block consisted of several words, but only accesses to each polygon block (not word) were counted. The same was also true of the remaining data structure. The following information was gathered:

Polygon Block Accesses-----336,156

Edge Block Accesses--------427,688

Vertex Array----------220,910

Planar Equation Array-------21,072

Total Accesses----------1,005,826

The number of accesses to memory was far greater than that required by the new scan line algorithm (47,030). Also, Warnock's algorithm requires that the complete object description be stored in fast memory, and not just those objects pertaining to the current scan line.

D. Output Buffering

Whenever a cathode ray tube (CRT) is being continually refreshed, the rate of moving the beam must remain constant if the displayed intensity is to be a function of the analog input intensity. That is, the X and Y deflection circuits must be changed at a constant rate. The output of the VSG

does not generate segments at a rate inversely proportional to the length of the segments. Therefore, a buffer for temporarily storing segments must be inserted between the VSG and the display.

In Figure 26 (Object 5: Cube2), the Y scan goes from the bottom to the top of the picture. The VSG can quickly determine the visibility of the bottom half of the picture but will require a great amount of time for the top half of the picture. The display, however, must spend the same amount of time on each half of the picture. Because of this, almost the entire bottom half of the picture would need to be buffered. On the other hand in Figure 23 (Object 2: E-S), the VSG runs at a fairly constant rate over the whole picture, and only a small amount of buffering would be required.

For the ten test objects, line 13 of Figure 33 shows the smallest number of segments that must be stored at one time in order to have a display running at thirty frames per second with a constant rate for the X and Y deflection of the CRT beam. The VSG was simulated to reference the polygon segment array every 200 nanoseconds. For objects which have a uniform distribution over the area, only a small buffer size was needed. For Cube2, which has a concentration of visible information in the upper right hand corner, a much larger buffer size was required.

CHAPTER VIII

CONCLUSION

The processor described can be built with equipment available today. The segment memory must be in the 200 nanosecond cycle range, and semiconductor memories are available in this range. Also, only a small memory is required since 18 to 50 segment blocks at most are needed at any one time for any of the ten test objects.

The algorithm has been simulated in Fortran IV on a PDP-10 at the Computer Science Department at the University of Utah. Other pictures have been taken to show how coloring and shading adds to the realism of objects. Figures 34-39 show various objects. Total computation time for generating and displaying the pictures is short. Cube1 (Object 4) required 30 seconds, and the church of Figure 35 containing 345 blocks (six polygons per block), required only 2.5 minutes. Figure 36 shows the back view of Figure 35 with the blocks randomly colored.

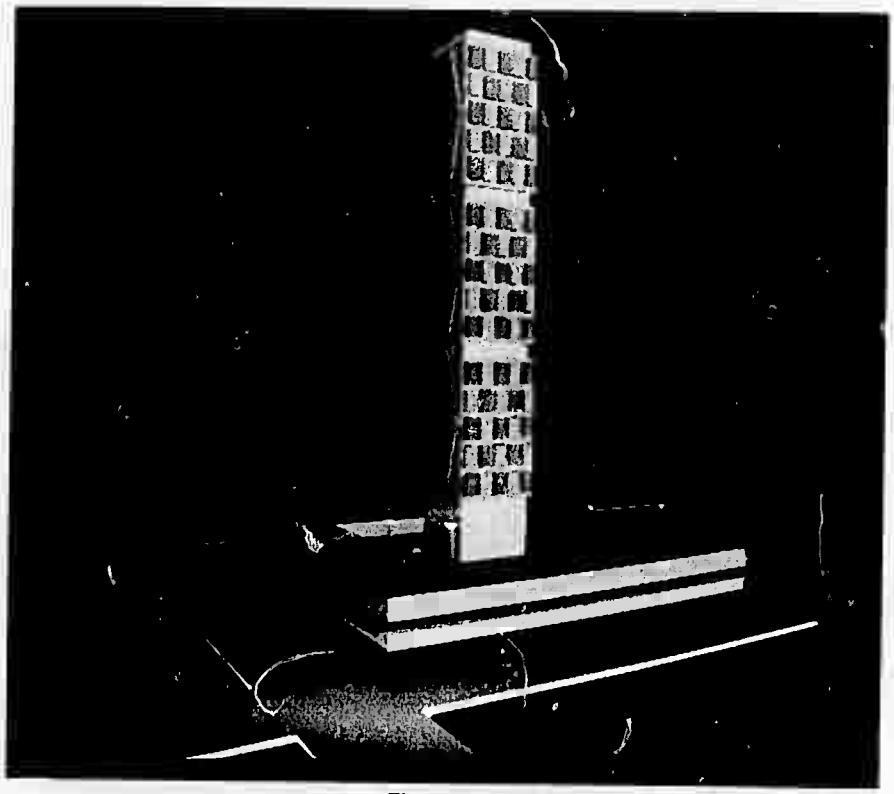


Figure 34
Office Structure

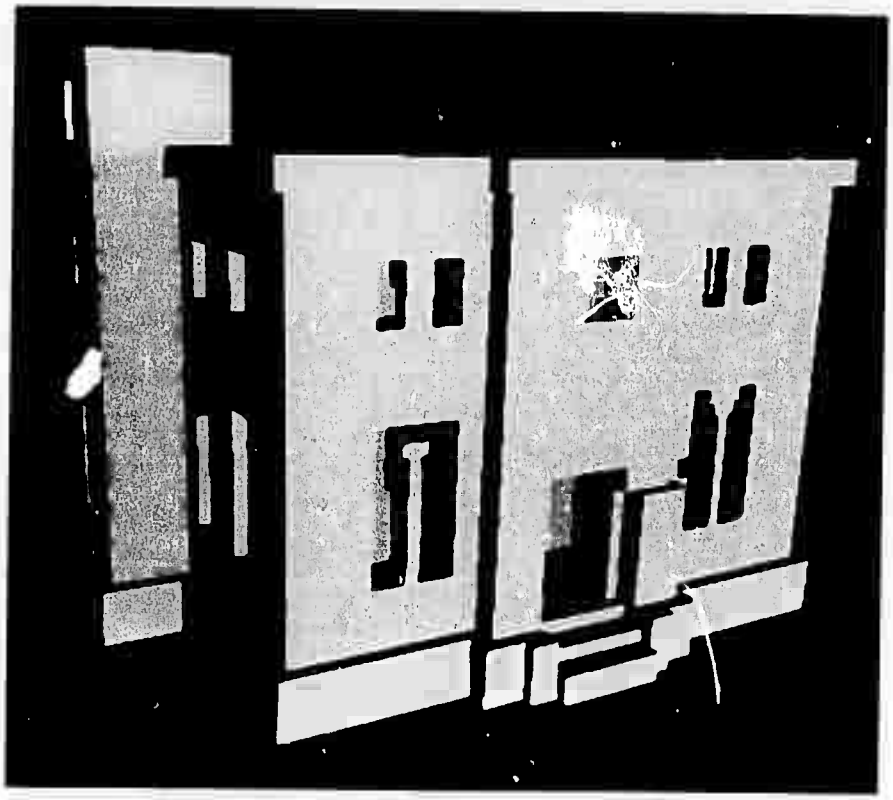


Figure 35
Church

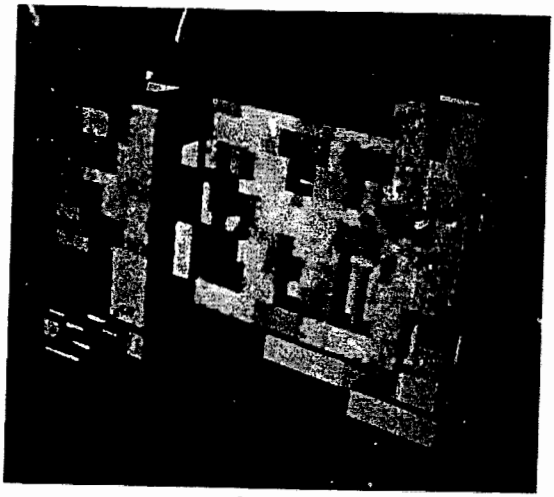


Figure 36
Rear View of Church with Randomly Colored Blocks

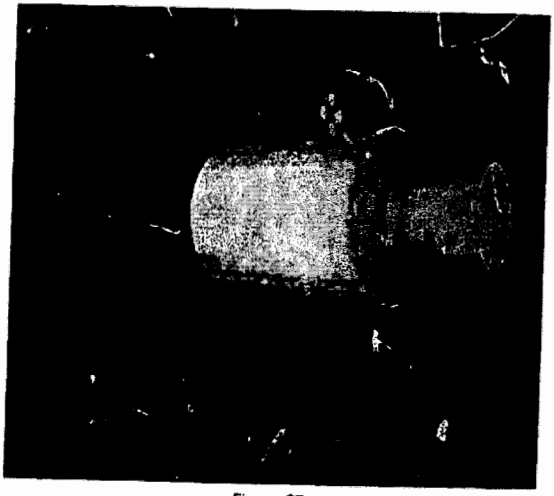


Figure 37
Apollo Command and Service Module

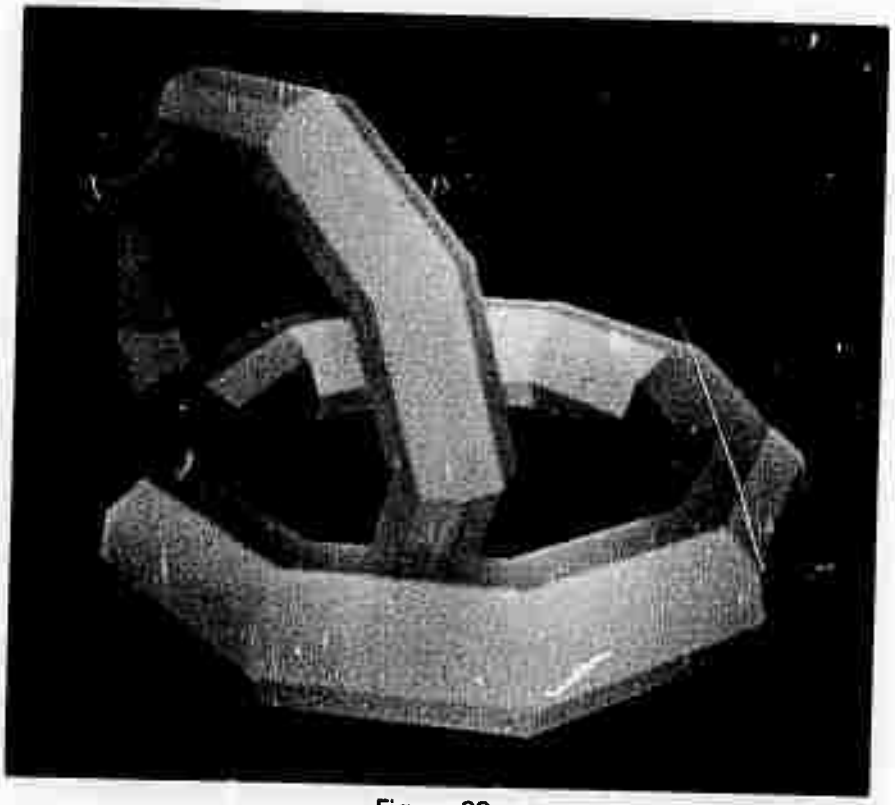


Figure 38
Tori

Figure 39
Randomly Colored Surface

BIBLIOGRAPHY

1. Roberts, L. G. "Machine Perception of Three-Dimensional Solids," Technical Report No. 315, Lincoln Laboratory, M.I.T., Cambridge, Mass., 22 May 1963.

2. Loutrel, P. P. "A Solution to the Hidden-Line Problem for Computer-Drawn Polyhedra," IEEE Transactions on Computers, C-19 [3], 205 March 1970.

3. Appel, A. "The Notion of Quantitative Invisibility and the Machine Rendering of Solids," ACM Conference Proc. 387 (1967).

4. Wylie, C., Romney, G., Evans, D. C., Erdahl, A. "Half-tone Perspective Drawings by Computer," AFIPS Proc. FJCC 31, 49 November 1967.

5. Romney, G. "Computer Assisted Assembly and Rendering of Solids," Computer Science, University of Utah, Salt Lake City, Utah, August 1969.

6. Warnock, J. "A Hidden Surface Algorithm for Computer Generated Halftone Pictures," Technical Report 4-15, Computer Science, University of Utah, Salt Lake City, Utah, June 1969.

7. Bouknight, W. J. "An Improved Procedure for Generation of Half-tone Computer Graphics Presentations," Report R-432, Coordinated Science Laboratory, University of Illinois, Urbana, Illinois, September 1969.

8. Sproull, R., Sutherland, I. E. "A Clipping Divider," AFIPS Proc. FJCC 33, 765 (1968).

9. McCallister, S., Sutherland, I. E. "Final Report on the Area Warnock Hidden Line Algorithm," Evans and Sutherland Computer Corporation, Salt Lake City, Utah, Internal Document, 12 February 1970.

APPENDIX I

LISTING OF PROGRAM

The hidden line program is called as a subroutine from a main program. VSG6 contains counters interspersed throughout the program for gathering statistics like those in Appendix II. VSG6 is written in FORTRAN IV.

Several subroutines are called by the program:

LDRPT(I,J) < loads the right half of J (sign extended) into I.

LDLPT(I,J) < loads the left half of J (sign extended) into I.

STRPT(I,J) < stores the right half of I into the right half of J. The left half of J remains undisturbed.

STLPT(I,J) < stores the right half of I into the left half of J. The right half of J remains undisturbed.

SHOW < displays the segments stored in the VISSEG array.

LSTSET(N) < initializes a free list structure with blocks of N words each.

GETBLK(I) < gets a block from the free list. I is the index of that block and is set by the subroutine.

RETBLK(I) < returns a block to the free list. I is the index of the block to be returned.

```
      SUBROUTINE HIDDEN(PIX,STAT)
      COMMON/FREE/EDGEST,DUM,POLYST
      COMMON/FREE1/Q1(4),FRAMEX,FRAMEY
      COMMON/FREE2/X(1000),Y(1000),Z(1000),CX(700),CY(700),
     1CZ(700),CD(700)
      IMPLICIT INTEGER (A-Z)
      REAL X,Y,Z,CX,CY,CZ,CD
      COMMON/SCOPE/VISSEG(512),BUCKY(512)
      DIMENSION EDGE(1),SEG(1),POLY(1)
      EQUIVALENCE (EDGEST,EDGE,SEG,POLY)
      DIMENSION ZS(10),SAM(3,2)
      DIMENSION PQ(16),Q(10,26),ADDS(20)
      PQL=16
      QL=26
C
C     PQ(1)=NUMBER OF TOTAL BLOCKS REQUIRED FOR
C          HIDDEN LINE WORK.
C     PQ(2)=MAXIMUM NUMBER OF TOTAL BLOCKS EVER USED AT ONE TIME.
C
C     PQ(3)=CURRENT NUMBER OF TOTAL BLOCKS AT A GIVEN TIME.
C          (USED FOR CALCULATING PQ(2).)
C     PQ(4)=TOTAL NUMBER OF EDGE BLOCKS IN FRAME.
C
C     PQ(5)=NUMBER OF EDGE BLOCKS WITH AT LEAST ONE OF THE
C          CONNECTED POLYGONS DRAWN CLOCKWISE.
C     PQ(6)=NUMBER OF THOSE EDGE BLOCKS OF PQ(5) WHOSE Y VALUE
C          OF THE BEGIN PT IS NOT THE SAME AS THE END PT Y VALUE.
C     PQ(7)=TOTAL NUMBER OF POLYGON BLOCKS IN THE FRAME.
C
C     PQ(8)=NUMBER OF POLYGON BLOCKS DRAWN CLOCKWISE.
C
C     PQ(9)=POINT DENSITY.
C
C     PQ(10)=NUMBER OF INVOLVED SCAN LINES.
C
C     PQ(11)=MEMORY REFERENCES FOR SEGMENT CREATOR.
C
C     PQ(12)=NANOSECONDS PER MEMORY REFERENCE FOR SEGMENT CREATOR.
C
C     PQ(13)=MEMORY REFERENCES FOR DEPTH CALCULATOR.
C
C     PQ(14)=NANOSECONDS PER MEMORY REFERENCE FOR DEPTH CALCULATOR.
C
C     PQ(15)=MEMORY REF. TOTAL PQ(11),PQ(13)
C
C     PQ(16)=NANOSECONDS FOR PQ(15).
C
C     ADDS(I)=NUMBER OF TIMES THE DEPTH TEST WAS SATISFIED IN.
```

```
C       Q COUNTERS
CC      Q(1,X)=TOTAL PER FRAME
CC      Q(2,X)=MAXIMUM REQUIRED OF A SCAN LINE
C C     Q(3,X)=AVERAGE OF TOTAL SCAN LINES. ALSO SCRATCH FOR Q(2,X)
C       Q(4,X)=REQUIRED FOR PRE-FRAME PROCESSING
CC      Q(5,X)=MAXIMUM REQUIRED FOR SCAN PREPARATION PROCESSING
CC      Q(6,X)=NANOSECONDS REQUIRED. ALSO SCRATCH FOR Q(5,X)
C C     Q(7,X)=MAXIMUM REQUIRED FOR SCAN DEPTH PROCESSING
C C     Q(8,X)=AVERAGE OF ACTIVE SCAN LINES. ALSO SCRATCH FOR Q(7,X)
CC      Q(9,X)=TOTAL FOR SCAN PREPARATION PROCESSING
CC      Q(10,X)=TOTAL FOR SCAN DEPTH PROCESSING
CC
CC
CC      Q(X,1)=NUMBER OF SLOPE CALCULATIONS.
CC      Q(X,2)=NUMBER OF INTERCEPT CALCULATIONS.
CC      Q(X,3)=NUMBER OF SAMPLE POINTS STORED FOR NEXT SCAN LINE.
CC      Q(X,4)=SUBDIVISIONS (NOT FROM INTERSECTING CASE).
CC      Q(X,5)=
CC      Q(X,6)=DEPTH SAMPLES REQUIRED.
CC      Q(X,7)=
CC      Q(X,8)=SAMPLE POINTS DELETED.
CC      Q(X,9)=
CC      Q(X,10)=OUTPUT SEGMENTS.
CC      Q(X,11)=INTERCEPT CALCULATIONS.
CC      Q(X,12)=INTERCEPT SUBDIVISIONS.
CC      Q(X,13)=OVERHEAD PIPELINE TIME.
CC      Q(X,14)=TIME WAITING FOR CLIPPER.
CC      Q(X,15)=READS FROM POLY.
CC      Q(X,16)=WRITES TO POLY.
CC      Q(X,17)=READS FROM EDGE.
CC      Q(X,18)=WRITES TO EDGE.
CC      Q(X,19)=READS FROM SEG
CC      Q(X,20)=WRITES TO SEG
CC      Q(X,21)=
CC      Q(X,22)=
CC      Q(X,23)=READS FROM FREE LIST(GETBLK)
CC      Q(X,24)=WRITES TO FREE LIST(RETBLK)
CC      Q(X,25)=READS FROM BUCKY
C       Q(X,26)=USED FOR SHADER
```

```
C       INITIALIZATION.
C
        DO 8 I=1,QL
        DO 8 J=1,10
8       Q(J,I)=0
        DO 9 I=1,PQL
9       PQ(I)=0
        DO 12 I=1,20
12      ADDS(I)=0
        DO 10 I=1,FRAMEY
10      BUCKY(I)=0
        CALL LSTSET(14)
        DEPTH=.TRUE.
        SAM2S=0
        SAM2X=FRAMEX
        SEGS2=0
        SEGL2=0
        POLYCH=0
        IMPLST=0
C       GO THROUGH ALL POLYGONS AND NOTE WHICH WAY EACH POLYGON
C       IS DRAWN (CLOCKWISE OR COUNTER CLOCKWISE) BY CHECKING
C       CZ OF PLANAR EQUATIONS AND MARK THE POLYGON BLOCK.
        POLYPT=POLYST
90      IF(POLYPT.EQ.0)GO TO 99
        POLY(POLYPT+1)=-1
        CALL LDRPT(INDEX,POLY(POLYPT+2))
        Q(1,15)=Q(1,15)+1
        PQ(7)=PQ(7)+1
        Q(1,16)=Q(1,16)+1
        IF(CZ(INDEX).LE.0)GO TO 95
        POLY(POLYPT+1)=0
        POLY(POLYPT+3)=0
        PQ(8)=PQ(8)+1
95      CALL LDRPT(POLYPT,POLY(POLYPT))
        GO TO 90
```

```
C       INITIALIZATION CONTINUED.                                          70
C       TAKE EACH EDGE AND PUT IN THE BUCKY GIVEN BY ITS
C       SMALLEST Y VALUE.  THIS IS THE Y-SORT OF EDGES.
99      EDGEPT=EDGEST
100     IF(EDGEPT.EQ.0)GO TO 200
        PQ(4)=PQ(4)+1
C
C       ENTER EACH EDGE IN BUCKY IF AT LEAST ONE OF THE
C       TWO POLYGONS IS DRAWN CLOCKWISE.
C
        CALL LDLPT(POLYL,EDGE(EDGEPT+2))
        CALL LDRPT(POLYR,EDGE(EDGEPT+2))
        Q(1,17)=Q(1,17)+1
        IF(POLYR.EQ.POLYL)GO TO 110
        IF(POLYL.EQ.0)GO TO 103
        IF(POLY(POLYL+1).EQ.0)GO TO 104
103     IF(POLY(POLYR+1).LT.0)GO TO 110
        Q(1,15)=Q(1,15)+1
104     CALL LDLPT(INDEX,EDGE(EDGEPT+1))
        YBEG=Y(INDEX)
        CALL LDRPT(INDEX,EDGE(EDGEPT+1))
        YEND=Y(INDEX)
        PQ(5)=PQ(5)+1
        IF(YBEG.EQ.YEND)GO TO 110
        PQ(6)=PQ(6)+1
        IF(YBEG.LT.YEND)GO TO 105
        I=YEND
        YEND=YBEG
        YBEG=I
105     YBEG=YBEG+1
        IF(YBEG.LE.0)GO TO 115
        IF(YEND.GE.FRAMEY)GO TO 115
        I=BUCKY(YBEG)
        BUCKY(YBEG)=EDGEPT
        Q(1,18)=Q(1,18)+1
        CALL STLPT(I,EDGE(EDGEPT))
110     CALL LDRPT(EDGEPT,EDGE(EDGEPT))
        GO TO 100
15      TYPE 116
        RETURN
16      FORMAT(' ERROR...OBJECT NOT IN BOUNDS OF FRAME!')
```

```
C       SCAN LINE COMPUTATION.
200     CONTINUE                                                        71
        DO 201 I=1,QL
201     Q(4,I)=Q(1,I)
        IY=0
204     IY=IY+1
        DO 202 I=1,QL
        Q(6,I)=Q(1,I)
        Q(9,I)=Q(9,I)-Q(1,I)
202     Q(3,I)=Q(1,I)
C       INITIALIZE ALL POINTERS.
        SEGXST=SEGS2
        SEGLST=SEGL2
        SEGS2=0
        SEGL2=0
        SAMIS=SAM2S
        SAMIL=SAM2L
        SAM2S=0
        SAMIX=SAM2X
        SAM2X=FRAMEX
        IF(IY.GT.FRAMEY)GO TO 230
        SEGCNT=0
C       SCAN PREPARATION PROCESSING.
C       GET EDGES FROM BUCKY WHICH ENTER ON THIS SCAN LINE
C       AND BUILD THE SEGMENT LIST (SEG).
        IF(BUCKY(IY).EQ.0)GO TO 230
        Q(1,25)=Q(1,25)+1
        EDGEPT=BUCKY(IY)
210     IF(EDGEPT.EQ.0)GO TO 230
        Q(1,17)=Q(1,17)+1
        CALL LDLPT(BEG,EDGE(EDGEPT+1))
        CALL LDRPT(END,EDGE(EDGEPT+1))
        YEND=Y(END)
        YBEG=Y(BEG)
        DELY=YBEG-YEND
        IF(DELY.EQ.0)GO TO 229
        IF(DELY.LT.0)GO TO 211
        I=BEG
        BEG=END
        END=I
        DELY=-DELY
211     IX=X(BEG)*262144.0
        Q(1,1)=Q(1,1)+1
        XSLOPE=((X(END)-X(BEG))/(Y(END)-Y(BEG)))*262144.0
        DEL=(IY-Y(BEG))*262144.0
        IX=IX+ZMUL(XSLOPE,DEL)
        CALL LDLPT(IXE,IX)
        IF(IXE.LE.0.OR.IXE.GT.FRAMEX)GO TO 115
        II=-1
        CALL LDRPT(POLYPT,EDGE(EDGEPT+2))
C       TWO POLYGONS PER EDGE ARE ALLOWED.
212     IF(POLYPT.EQ.0)GO TO 228
        Q(1,15)=Q(1,15)+1
        IF(POLY(POLYPT+1).EQ.-1)GO TO 228
        Q(1,16)=Q(1,16)+1
        IF(POLY(POLYPT+1).LT.0)GO TO 213
        POLY(POLYPT+1)=POLYCH
        POLYCH=POLYPT
        CALL STLPT(-1,POLY(POLYPT+1))
213     SEGPT=POLY(POLYPT+3)
        PREV=0
        YEND2P=-1
```

```
214     IF(SEGPT.EQ.0)GO TO 220
        Q(1,19)=Q(1,19)+1
        CALL LDRPT(YEND2,SEG(SEGPT+2))
        CALL LDLPT(YEND1,SEG(SEGPT+2))
        IF(YEND1.GE.0)GO TO 217
        TE1=IX-SEG(SEGPT+3)-SEG(SEGPT+4)
        IF(TE1.EQ.0)TE1=XSLOPE-SEG(SEGPT+4)
        IF(TE1.LT.0)GO TO 220
        IF(YEND2.GE.0)GO TO 218
        TE2=IX-SEG(SEGPT+5)-SEG(SEGPT+6)
        IF(TE2.EQ.0)TE2=XSLOPE-SEG(SEGPT+6)
        IF(TE2.LT.0)GO TO 223
        GO TO 218
217     IF(YEND2.GE.0)GO TO 218
        TE2=IX-SEG(SEGPT+5)-SEG(SEGPT+6)
        IF(TE2.EQ.0)TE2=XSLOPE-SEG(SEGPT+6)
        IF(TE2.GE.0)GO TO 218
        MODE=0
        PREV=SEGPT
        GO TO 227
218     YEND2P=YEND2
        PREV=SEGPT
        CALL LDRPT(SEGPT,SEG(SEGPT+1))
        GO TO 214
220     MODE=2
        IF(YEND2P.GE.0)GO TO 227
        FROM=0
        GO TO 226
```

```
223     FROM=-1
        PREV=SEGPT
        CALL LDRPT(SEGPT,SEG(SEGPT+1))
        GO TO 226
224     SEG(I+5)=SEG(PREV+5)
        SEG(I+6)=SEG(PREV+6)
        SEG(I+9)=SEG(PREV+9)
        SEG(I+10)=SEG(PREV+10)
        Q(1,20)=Q(1,20)+1
        CALL STRPT(YEND2,SEG(I+2))
        MODE=2
        GO TO 227
226     CALL GETBLK(I)
        Q(1,23)=Q(1,23)+1
        PQ(1)=PQ(1)+1
        PQ(3)=PQ(3)+1
        IF(PQ(3).GT.PQ(2))PQ(2)=PQ(3)
        CALL STRPT(SEGPT,SEG(I+1))
        IF(PREV.NE.0)CALL STRPT(I,SEG(PREV+1))
        IF(PREV.NE.0)Q(1,20)=Q(1,20)+1
        IF(PREV.EQ.0)POLY(POLYPT+3)=I
        SEG(I)=-1
        SEG(I+2)=0
        SEG(I+11)=0
        CALL STLPT(POLYPT,SEG(I+1))
        IF(FROM.NE.0)GO TO 224
        PREV=I
227     SEG(PREV+3+MODE)=IX-XSLOPE
        SEG(PREV+4+MODE)=XSLOPE
        Q(1,20)=Q(1,20)+1
        IF(MODE.EQ.0)CALL STLPT(DELY,SEG(PREV+2))
        IF(MODE.NE.0)CALL STRPT(DELY,SEG(PREV+2))
        SEG(PREV+8+MODE)=((Z(END)-Z(BEG))/(Y(END)-Y(BEG)))*262144.0
        SEG(PREV+7+MODE)=Z(BEG)*262144.0
        SEG(PREV+7+MODE)=SEG(PREV+7+MODE)+ZMUL(SEG(PREV+8+MODE),DEL)
        SEG(PREV+7+MODE)=SEG(PREV+7+MODE)-SEG(PREV+8+MODE)
228     CALL LDLPT(POLYPT,EDGE(EDGEPT+2))
        II=II+1
        IF(II.EQ.0)GO TO 212
229     CALL LDLPT(EDGEPT,EDGE(EDGEPT))
        GO TO 210
```

```
C       SEGMENT PACKER AND SEGMENT ELIMINATOR.
230     IF (POLYCH.EQ.0) GO TO 242
        Q(1,15)=Q(1,15)+1
        Q(1,16)=Q(1,16)+1
        CALL STLPT(0,POLY(POLYCH+1))
        NEXT=POLY(POLYCH+3)
        SEGPT=0
231     IF (NEXT.EQ.0) GO TO 240
        PREV=SEGPT
        SEGPT=NEXT
        Q(1,19)=Q(1,19)+1
        CALL LDRPT(NEXT,SEG(SEGPT+1))
        IF (SEG(SEGPT+2).NE.0) GO TO 233
        IF(PREV.NE.0)Q(1,20)=Q(1,20)+1
        IF(PREV.NE.0)CALL STRPT(NEXT,SEG(PREV+1))
        IF(PREV.EQ.0)POLY(POLYCH+3)=NEXT
        Q(1,24)=Q(1,24)+1
        PQ(3)=PQ(3)-1
        CALL RETBLK(SEGPT)
        SEGPT=PREV
        GO TO 231
233     NEXT1=NEXT
        CALL LDRPT(YEND2,SEG(SEGPT+2))
        IF (YEND2.GE.0) GO TO 237
        CALL LDLPT(YEND1,SEG(SEGPT+2))
        IF(YEND1.LT.0)GO TO 2395
        SEG(SEGPT+3)=SEG(SEGPT+5)
        SEG(SEGPT+4)=SEG(SEGPT+6)
        SEG(SEGPT+7)=SEG(SEGPT+9)
        SEG(SEGPT+8)=SEG(SEGPT+10)
        CALL STLPT(YEND2,SEG(SEGPT+2))
```

```
237     IF(NEXT1.EQ.0)GO TO 241
        CALL LDLPT(YEND1,SEG(NEXT1+2))
        Q(1,19)=Q(1,19)+1
        IF (YEND1.GE.0) GO TO 238
        Q(1,20)=Q(1,20)+2
        CALL STRPT(YEND1,SEG(SEGPT+2))
        CALL STLPT(0,SEG(NEXT1+2))
        SEG(SEGPT+5)=SEG(NEXT1+3)
        SEG(SEGPT+6)=SEG(NEXT1+4)
        SEG(SEGPT+9)=SEG(NEXT1+7)
        SEG(SEGPT+10)=SEG(NEXT1+8)
        CALL LDLPT(S1,SEG(NEXT1))
        CALL LDRPT(S2,SEG(NEXT1))
        IF(S1.NE.0)CALL STRPT(S2,SEG(S1))
        IF(S1.EQ.0)SEGXST=S2
        IF(NEXT1.NE.SEGLST)CALL STLPT(S1,SEG(S2))
        IF(NEXT1.EQ.SEGLST)SEGLST=S1
        Q(1,20)=Q(1,20)+1
        SEG(NEXT1)=-1
        GO TO 2395
238     CALL LDRPT(YEND2,SEG(NEXT1+2))
        IF (YEND2.GE.0) GO TO 239
        Q(1,20)=Q(1,20)+2
        CALL STRPT(YEND2,SEG(SEGPT+2))
        SEG(NEXT1+2)=0
        SEG(SEGPT+5)=SEG(NEXT1+5)
        SEG(SEGPT+6)=SEG(NEXT1+6)
        SEG(SEGPT+9)=SEG(NEXT1+9)
        SEG(SEGPT+10)=SEG(NEXT1+10)
        GO TO 2395
239     CALL LDRPT(NEXT1,SEG(NEXT1+1))
        GO TO 237
2395    IF(SEG(SEGPT).NE.-1)GO TO 231
        CALL LDLPT(IXE,SEG(SEGPT+3)+SEG(SEGPT+4))
        S1=PREV
2396    IF(S1.NE.0)CALL LDRPT(S2,SEG(S1))
        IF(S1.EQ.0)S2=SEGXST
        IF(S1.EQ.SEGLST)S2=0
        Q(1,19)=Q(1,19)+1
        IF(S2.EQ.0)GO TO 2397
        CALL LDLPT(IX,SEG(S2+3)+SEG(S2+4))
        IF(IX.GE.IXE)GO TO 2397
        S1=S2
        GO TO 2396
2397    IF(S2.NE.0)SEG(SEGPT)=S2
        Q(1,20)=Q(1,20)+1
        CALL STLPT(S1,SEG(SEGPT))
        IF(S1.NE.0)CALL STRPT(SEGPT,SEG(S1))
        IF(S1.EQ.0)SEGXST=SEGPT
        IF(S2.NE.0)CALL STLPT(SEGPT,SEG(S2))
        IF(S2.EQ.0)SEGLST=SEGPT
        GO TO 231
240     POLYCH=POLY(POLYCH+1)
        GO TO 230
241     PAUSE 'UNCLOSED POLYGON'
        SEG(SEGPT+5)=SEG(SEGPT+3)
        SEG(SEGPT+6)=SEG(SEGPT+4)
        CALL STRPT(0,SEG(SEGPT+2))
        GO TO 2395
```

```
C       DEPTH SORTER.
242     CONTINUE
        DO 276 I=1,QL
        Q(6,I)=Q(1,I)-Q(6,I)
        IF(Q(5,I).LT.Q(6,I))Q(5,I)=Q(6,I)
        Q(9,I)=Q(9,I)+Q(1,I)
        Q(10,I)=Q(10,I)-Q(1,I)
276     Q(8,I)=Q(1,I)
        IF(IY.GT.FRAMEY)GO TO 498
        SAM(1,2)=1
        SAM(2,2)=0
        CALL LDLPT(SEGPT,IMPLST)
278     IF(SEGPT.EQ.0)GO TO 279
        NEXT=SEG(SEGPT)
        CALL RETBLK(SEGPT)
        PQ(3)=PQ(3)-1
        Q(1,24)=Q(1,24)+1
        SEGPT=NEXT
        GO TO 278
279     IMPLST=IMPLST*262144
        SEGACT=0
C               SAMPLE SPAN GENERATOR.
281     SAM(1,1)=SAM(1,2)+1
        SAM(2,1)=SAM(2,2)
        SAM(3,1)=SAM(3,2)
        SAM(2,2)=0
        IF(SAMIX.GE.SAM(1,1))GO TO 282
        SAMIX=FRAMEX
        IF(SAMIS.EQ.SAMIL)GO TO 282
        CALL LDLPT(SAMIX,SEG(SAMIS))
        CALL LDRPT(SAMIS,SEG(SAMIS))
282     SAM(1,2)=SAMIX
299     ZS(1)=0
        FROM=0
        SEGPT=SEGACT
        SEGOUT=0
        PREV=0
```

```
C       CHECK SEGMENTS FROM THE CURRENT ACTIVE LIST.
301     IF (SEGPT.EQ.0) GO TO 304
        NUMREF=-Q(1,19)-Q(1,20)-Q(1,13)
        Q(1,19)=Q(1,19)+1
        NEXT=SEG(SEGPT+11)
        XLEFT=SEG(SEGPT+3)
        XRIGHT=SEG(SEGPT+5)
        ZLEFT=SEG(SEGPT+7)
        ZRIGHT=SEG(SEGPT+9)
        CALL LDLPT(IXE,XLEFT)
        CALL LDLPT(IXX,XRIGHT)
        IF(IXX.LE.SAM(1,2))GO TO 303
        PREV=SEGPT
        IF(IXE.GE.SAM(1,2))GO TO 335
        GO TO 315
303     CONTINUE
        Q(1,20)=Q(1,20)+1
        IF(PREV.NE.0)SEG(PREV+11)=NEXT
        IF(PREV.EQ.0)SEGACT=NEXT
        IF (IXX.LT.SAM(1,1)) GO TO 335
        Q(1,20)=Q(1,20)+1
        SEG(SEGPT+11)=SEGOUT
        IF (SEGOUT.EQ.0) SEGLO=SEGPT
        SEGOUT=SEGPT
        GO TO 315
C       CHECK NEW SEGMENTS FROM THE X-SORT LIST. ALSO
C       INCREMENT THE X,Y,Z VALUES AND INSERT THE SEGMENT BLOCK
C       IN THE X-SORT LIST FOR THE NEXT SCAN LINE.
304     SEGPT=SEGXST
        IF(SEGPT.EQ.0)GO TO 350
        NUMREF=-Q(1,19)-Q(1,20)-Q(1,13)
        Q(1,19)=Q(1,19)+1
        CALL LDLPT(IXE,SEG(SEGPT+3)+SEG(SEGPT+4))
        IF(IXE.GE.SAM(1,2))GO TO 350
        FROM=-1
        CALL LDRPT(SEGXST,SEG(SEGPT))
        IF(SEGPT.EQ.SEGLST)SEGXST=0
        Q(1,2)=Q(1,2)+1
        SEG(SEGPT+3)=SEG(SEGPT+3)+SEG(SEGPT+4)
        SEG(SEGPT+5)=SEG(SEGPT+5)+SEG(SEGPT+6)
        SEG(SEGPT+7)=SEG(SEGPT+7)+SEG(SEGPT+8)
        SEG(SEGPT+9)=SEG(SEGPT+9)+SEG(SEGPT+10)
        XLEFT=SEG(SEGPT+3)
        XRIGHT=SEG(SEGPT+5)
        ZLEFT=SEG(SEGPT+7)
        ZRIGHT=SEG(SEGPT+9)
        CALL LDLPT(YEND1,SEG(SEGPT+2))
        CALL LDRPT(YEND2,SEG(SEGPT+2))
        YEND1=YEND1+1
        YEND2=YEND2+1
        CALL STLPT(YEND1,SEG(SEGPT+2))
        CALL STRPT(YEND2,SEG(SEGPT+2))
```

```
      IF (SEG(SEGPT+11).GE.0) GO TO 309
      IF (YEND2.GE.0) GO TO 308
      IF (IXE+1.NE.SAM(1,1)) GO TO 308
      CALL LDLPT(IX,SEG(SEGPT+3)+SEG(SEGPT+4))
      IF (IX.LE.0.OR.IX.GT.FRAMEX) GO TO 308
      SAM(3,1)=SEGPT+12
      SAM(2,1)=IX
      FM=-1
      GO TO 3091
308   CALL RETBLK(SEGPT)
      PQ(3)=PQ(3)-1
      Q(1,24)=Q(1,24)+1
      GO TO 335
309   MODE=0
      SEG(SEGPT)=-1
      IF (YEND1.GE.0) GO TO 310
      MODE=-1
      CALL LDLPT(IX,SEG(SEGPT+3)+SEG(SEGPT+4))
      IF (IX.LE.0.OR.IX.GT.FRAMEX) GO TO 115
      FM=0
3091  S2=0
      S1=SEGL2
3092  IF(S1.EQ.0)GO TO 3094
      CALL LDLPT(IX1,SEG(S1+3)+SEG(S1+4))
      IF(IX.GE.IX1)GO TO 3094
      S2=S1
      CALL LDLPT(S1,SEG(S1))
      Q(1,19)=Q(1,19)+1
      GO TO 3092
3094  IF(S2.NE.0)SEG(SEGPT)=S2
      Q(1,20)=Q(1,20)+1
      CALL STLPT(S1,SEG(SEGPT))
      IF(S2.NE.0)CALL STLPT(SEGPT,SEG(S2))
      IF(S2.EQ.0)SEGL2=SEGPT
      IF(S1.NE.0)CALL STRPT(SEGPT,SEG(S1))
      IF(S1.EQ.0)SEGS2=SEGPT
      IF(S2.NE.0)Q(1,20)=Q(1,20)+1
      IF(FM)335,310,364
310   MODE=-MODE
      IF (YEND2.GE.0) GO TO 311
      MODE=-MODE
      CALL LDLPT(IX,SEG(SEGPT+5)+SEG(SEGPT+6))
      IF (IX.LE.0.OR.IX.GT.FRAMEX) GO TO 115
311   IF (MODE.LT.0) GO TO 312
C     IF EITHER OF THE EDGES OF THE SEGMENT EXIT ON THIS
C     SCAN LINE (MODE), PUT THE CORRESPONDING POLYGON IN
C     THE POLYGON CHANGING LIST.
      CALL LDLPT(POLYPT,SEG(SEGPT+1))
      Q(1,15)=Q(1,15)+1
      IF (POLY(POLYPT+1).LT.0) GO TO 312
      Q(1,16)=Q(1,16)+1
      POLY(POLYPT+1)=POLYCH
      POLYCH=POLYPT
      CALL STLPT(-1,POLY(POLYPT+1))
312   CALL LDLPT(IXX,XRIGHT)
      IF (IXE.GE.IXX) GO TO 335
      IF (IXX.GT.SAM(1,2)) GO TO 314
      SEG(SEGPT+11)=SEGOUT
      IF (SEGOUT.EQ.0) SEGLO=SEGPT
      SEGOUT=SEGPT
      GO TO 315
314   SEG(SEGPT+11)=SEGACT
      SEGACT=SEGPT
```

```
315     CONTINUE
        Q(1,6)=Q(1,6)+1
        IXLEFT=IXE
        IF(IXE.LT.SAM(1,1))IXLEFT=IXX
        CALL LDLPT(YEND1,SEG(SEGPT+2))
        IF(YEND1.GE.0)GO TO 316
        IF(IXE+1.NE.SAM(1,1))GO TO 316
        SAM(3,1)=SEGPT+12
        CALL LDLPT(SAM(2,1),SEG(SEGPT+3)+SEG(SEGPT+4))
316     CALL LDRPT(YEND2,SEG(SEGPT+2))
        IF(YEND2.GE.0)GO TO 317
        IF(IXX.NE.SAM(1,2))GO TO 317
        SAM(3,2)=SEGPT+13
        CALL LDLPT(SAM(2,2),SEG(SEGPT+5)+SEG(SEGPT+6))
C       SIMULATION OF LOADING AND RUNNING THE CLIPPER ARTHIMETIC
C       UNIT FOR DEPTH COMPARISONS.
C       ADDITION TIME ONE.
317     XLTEST=XLEFT.AND.(.NOT.262143)
        XRTEST=XRIGHT.AND.(.NOT.262143)
        NUMADD=1
        IF(FROM.NE.0)NUMADD=NUMADD+1
        DELNEW=(XRTEST-XLTEST)/262144
        IF(.NOT.DEPTH)DELNEW=DELNEW*1024
        IF((XLTEST-SAM(1,1)*262144).LT.0)XLTEST=SAM(1,1)*262144
        IF((XRTEST-SAM(1,2)*262144).GE.0)XRTEST=SAM(1,2)*262144
        ADJNEW=.FALSE.
        IF(ZLEFT.LT.ZRIGHT)ADJNEW=.TRUE.
        IF(ZS(1).EQ.0)GO TO 331
        ABLLE=.FALSE.
        ABLGE=.FALSE.
        ABRLE=.FALSE.
        ABRGE=.FALSE.
        IF(XLTEST.LE.ZS(6))ABLLE=.TRUE.
        IF(XLTEST.GE.ZS(6))ABLGE=.TRUE.
        IF(XRTEST.LE.ZS(7))ABRLE=.TRUE.
        IF(XRTEST.GE.ZS(7))ABRGE=.TRUE.
        IF(((.NOT.ABLGE).AND.(.NOT.ABRGE)).OR.((.NOT.ABLLE).AND.
       1(.NOT.ABRLE)))GO TO 329
        XLCLIP=XLTEST
        IF(ABLLE)XLCLIP=ZS(6)
        XRCLIP=XRTEST
        IF(ABRGE)XRCLIP=ZS(7)
        DEL=DELNEW
        IF(DELNEW.LT.ZSDEL)DEL=ZSDEL
```

```
      XAMXL=XLEFT-XLCLIP                                                  80
      XBMXL=XRIGHT-XLCLIP
      XAMXR=XLEFT-XRCLIP
      XBMXR=XRIGHT-XRCLIP
      ZAL=ZLEFT
      ZBL=ZRIGHT
      ZAR=ZLEFT
      ZBR=ZRIGHT
      IF(ADJNEW)GO TO 320
      ZBL=ZLEFT
      ZAL=ZRIGHT
      ZBR=ZLEFT
      ZAR=ZRIGHT
320   XCMXL=ZS(2)-XLCLIP
      XDMXL=ZS(3)-XLCLIP
      XCMXR=ZS(2)-XRCLIP
      XDMXR=ZS(3)-XRCLIP
      IF(ZS(1)-2.GE.0)GO TO 321
      ZCL=ZS(4)
      ZDL=ZS(5)
      ZCR=ZS(4)
      ZDR=ZS(5)
      ADJOLD=ZSADJ
      GO TO 323
321   ADJOLD=.NOT.ADJNEW
      IF(ADJNEW)GO TO 322
      ZCL=ZS(4)
      ZDL=ZS(4)
      ZCR=ZS(5)
      ZDR=ZS(5)
      GO TO 323
322   ZCL=ZS(5)
      ZDL=ZS(5)
      ZCR=ZS(4)
      ZDR=ZS(4)
```

```
C
323     CLIP STATE *** ONE ADD TIME EACH PASS.
        ABBCKL=.FALSE.
        ABBCKR=.FALSE.
        CDBCKL=.FALSE.
        CDBCKR=.FALSE.
        DELZ=.FALSE.
        NUMADD=NUMADD+1
        XHOLDL=(XAMXL+XBMXL)/2
        ZHOLDL=(ZAL+ZBL)/2
        XHOLDR=(XAMXR+XBMXR)/2
        ZHOLDR=(ZAR+ZBR)/2
        XTEMPL=(XCMXL+XDMXL)/2
        ZTEMPL=(ZCL+ZDL)/2
        XTEMPR=(XCMXR+XDMXR)/2
        ZTEMPR=(ZCR+ZDR)/2
        DEL=DEL/2
        IF(ZAL-ZDL.GE.0)ABBCKL=.TRUE.
        IF(ZCL-ZBL.GE.0)CDBCKL=.TRUE.
        IF(ZAR-ZDR.GE.0)ABBCKR=.TRUE.
        IF(ZCR-ZBR.GE.0)CDBCKR=.TRUE.
        IF(DEL.EQ.0)DELZ=.TRUE.
        LOG=((.NOT.ABLGE.OR..NOT.ABRLE).AND.((CDBCKL.AND..NOT.ABBCKR
       1.AND..NOT.CDBCKR).OR.(.NOT.ABBCKL.AND..NOT.CDBCKL.AND.CDBCKR)
       2.OR.(.NOT.ABBCKL.AND..NOT.CDBCKL.AND..NOT.ABBCKR)))
        LOG=LOG.OR.((.NOT.ABLLE.OR..NOT.ABRGE).AND.((ABBCKL.AND..NOT.
       1ABBCKR.AND..NOT.CDBCKR).OR.(.NOT.ABBCKL.AND..NOT.CDBCKL.AND.
       2ABBCKR).OR.(.NOT.ABBCKL.AND..NOT.CDBCKL.AND..NOT.CDBCKR)))
        LOG=LOG.OR.((.NOT.(ABBCKL.AND.ABBCKR).AND..NOT.(CDBCKL.AND.
       1CDBCKR)).AND.(ABLGE.AND.ABRLE.AND.ABLLE.AND.ABRGE))
        JCLIP=LOG.AND..NOT.DELZ
        LOG=((ABLGE.AND.ABRLE).AND.((ABBCKL.AND.ABBCKR).OR.(ABBCKL.AND.
       1.NOT.CDBCKR.AND.DELZ).OR.(ABBCKR.AND..NOT.CDBCKL.AND.DELZ)
       2.OR.(.NOT.CDBCKL.AND..NOT.CDBCKR.AND.DELZ)))
        J0BOX=LOG.OR.(DELZ.AND..NOT.ABBCKL.AND..NOT.CDBCKL
       1.AND..NOT.ABBCKR.AND..NOT.CDBCKR.AND.ABLGE.AND.ABRLE)
        LOG=(ABLLE.AND.ABRGE).AND..NOT.(ABLGE.AND.ABRLE.AND.((ABBCKL.
       1AND.ABBCKR).OR.(ABBCKL.AND..NOT.CDBCKR).OR.(ABBCKR
       2.AND..NOT.CDBCKL)))
        LOG=LOG.AND.((CDBCKL.AND.CDBCKR).OR.(CDBCKL.AND.
       1.NOT.ABBCKR.AND.DELZ).OR.(CDBCKR.AND..NOT.ABBCKL.AND.DELZ))
        J1BOX=LOG.OR.(DELZ.AND..NOT.ABBCKL.AND..NOT.CDBCKL
       1.AND..NOT.ABBCKR.AND..NOT.CDBCKR.AND.((ABLLE
       2.AND..NOT.ABRLE).OR.(.NOT.ABLGE.AND.ABRGE)))
        LOG=(DELZ.AND.((ABBCKL.AND..NOT.CDBCKL.AND..NOT.ABBCKR.AND.
       1CDBCKR).OR.(.NOT.ABBCKL.AND.CDBCKL.AND.ABBCKR
       2.AND..NOT.CDBCKR)))
        LOG=LOG.OR.((.NOT.ABLGE.OR..NOT.ABRLE).AND.((ABBCKL
       1.AND..NOT.CDBCKL).OR.(ABBCKR.AND..NOT.CDBCKR)))
        LOG=LOG.OR.((.NOT.ABLLE.OR..NOT.ABRGE).AND.((CDBCKL
       1.AND..NOT.ABBCKL).OR.(CDBCKR.AND..NOT.ABBCKR)))
        JBOXES=LOG.OR.(DELZ.AND..NOT.ABBCKL.AND..NOT.CDBCKL
       1.AND..NOT.ABBCKR.AND..NOT.CDBCKR.AND.((.NOT.ABLLE
       2.AND..NOT.ABRLE).OR.(.NOT.ABLGE.AND..NOT.ABRGE)))
        NUMCNT=0
        IF(JCLIP)NUMCNT=NUMCNT+1
        IF(J1BOX)NUMCNT=NUMCNT+1
        IF(JBOXES)NUMCNT=NUMCNT+1
        IF(J0BOX)NUMCNT=NUMCNT+1
        IF(NUMCNT.NE.1)PAUSE
        IF(JCLIP)GO TO 325
        IF(J1BOX)GO TO 331
        IF(JBOXES)GO TO 329
        IF(J0BOX)GO TO 335
```

```
325     IF(XHOLDL.GE.0)XBMXL=XHOLDL
        IF(XHOLDL.GE.0.AND.ADJNEW)ZBL=ZHOLDL
        IF(XHOLDL.GE.0.AND.(.NOT.ADJNEW))ZAL=ZHOLDL
        IF(XHOLDL.LT.0)XAMXL=XHOLDL
        IF(XHOLDL.LT.0.AND.ADJNEW)ZAL=ZHOLDL
        IF(XHOLDL.LT.0.AND.(.NOT.ADJNEW))ZBL=ZHOLDL
        IF(XHOLDR.GE.0)XBMXR=XHOLDR
        IF(XHOLDR.GE.0.AND.ADJNEW)ZBR=ZHOLDR
        IF(XHOLDR.GE.0.AND.(.NOT.ADJNEW))ZAR=ZHOLDR
        IF(XHOLDR.LT.0)XAMXR=XHOLDR
        IF(XHOLDR.LT.0.AND.ADJNEW)ZAR=ZHOLDR
        IF(XHOLDR.LT.0.AND.(.NOT.ADJNEW))ZBR=ZHOLDR
        IF(XTEMPL.GE.0)XDMXL=XTEMPL
        IF(XTEMPL.GE.0.AND.ADJOLD)ZDL=ZTEMPL
        IF(XTEMPL.GE.0.AND.(.NOT.ADJOLD))ZCL=ZTEMPL
        IF(XTEMPL.LT.0)XCMXL=XTEMPL
        IF(XTEMPL.LT.0.AND.ADJOLD)ZCL=ZTEMPL
        IF(XTEMPL.LT.0.AND.(.NOT.ADJOLD))ZDL=ZTEMPL
        IF(XTEMPR.GE.0)XDMXR=XTEMPR
        IF(XTEMPR.GE.0.AND.ADJOLD)ZDR=ZTEMPR
        IF(XTEMPR.GE.0.AND.(.NOT.ADJOLD))ZCR=ZTEMPR
        IF(XTEMPR.LT.0)XCMXR=XTEMPR
        IF(XTEMPR.LT.0.AND.ADJOLD)ZCR=ZTEMPR
        IF(XTEMPR.LT.0.AND.(.NOT.ADJOLD))ZDR=ZTEMPR
        GO TO 323
325     DEL=ZSDEL
        XAMXL=XLCLIP
        XBMXL=XRCLIP
        ZAL=ZAL-ZCL
        ZBL=ZAR-ZCR
327     ZHOLDL=(ZAL+ZBL)/2
        XHOLDL=(XAMXL+XBMXL)/2
        NUMADD=NUMADD+1
        DEL=DEL/2
        IF(DEL.EQ.0)GO TO 335
        IF(ZAL.XOR.ZHOLDL.GE.0)XAMXL=XHOLDL
        IF(ZBL.XOR.ZHOLDL.GE.0)XBMXL=XHOLDL
        IF(ZAL.XOR.ZHOLDL.GE.0)ZAL=ZHOLDL
        IF(ZBL.XOR.ZHOLDL.GE.0)ZBL=ZHOLDL
        GO TO 327
```

```
C       EXPAND BOX TO INCLUDE OLD BOX AND NEW LINE CLIPPED.
329     ZS(1)=ZS(1)+1
        IF(.NOT.DEPTH)GO TO 326
        IF(ABRLE.AND.ABRGE.AND.ABLLE.AND.ABLGE)GO TO 3295
        IF(ZLEFT-ZS(4).LT.0.AND.ADJNEW)ZS(4)=ZLEFT
        IF(ZRIGHT-ZS(4).LT.0.AND.(.NOT.ADJNEW))ZS(4)=ZRIGHT
        IF(ZLEFT-ZS(5).GE.0.AND.(.NOT.ADJNEW))ZS(5)=ZLEFT
        IF(ZRIGHT-ZS(5).GE.0.AND.ADJNEW)ZS(5)=ZRIGHT
        GO TO 330
3295    IF(ADJNEW.AND.ADJOLD.AND.CDBCKL)ZS(4)=ZAL
        IF(ADJNEW.AND.ADJOLD.AND.ABBCKL)ZS(4)=ZCL
        IF(ADJNEW.AND..NOT.ADJOLD.AND.ZAL.LT.ZCR)ZS(4)=ZAL
        IF(ADJNEW.AND..NOT.ADJOLD.AND.ZAL.GE.ZCR)ZS(4)=ZCR
        IF(.NOT.ADJNEW.AND.ADJOLD.AND.ZAR.LT.ZCL)ZS(4)=ZAR
        IF(.NOT.ADJNEW.AND.ADJOLD.AND.ZAR.GE.ZCL)ZS(4)=ZCL
        IF(.NOT.ADJNEW.AND..NOT.ADJOLD.AND.CDBCKR)ZS(4)=ZAR
        IF(.NOT.ADJNEW.AND..NOT.ADJOLD.AND.ABBCKR)ZS(4)=ZCR
        IF(ADJNEW.AND.ADJOLD.AND.CDBCKR)ZS(5)=ZDR
        IF(ADJNEW.AND.ADJOLD.AND.ABBCKR)ZS(5)=ZBR
        IF(ADJNEW.AND..NOT.ADJOLD.AND.ZBR.LT.ZDL)ZS(5)=ZDL
        IF(ADJNEW.AND..NOT.ADJOLD.AND.ZBR.GE.ZDL)ZS(5)=ZBR
        IF(.NOT.ADJNEW.AND.ADJOLD.AND.ZBL.LT.ZDR)ZS(5)=ZDR
        IF(.NOT.ADJNEW.AND.ADJOLD.AND.ZBL.GE.ZDR)ZS(5)=ZBL
        IF(.NOT.ADJNEW.AND..NOT.ADJOLD.AND.CDBCKL)ZS(5)=ZDL
        IF(.NOT.ADJNEW.AND..NOT.ADJOLD.AND.ABBCKL)ZS(5)=ZBL
330     IF(ABLLE)ZS(6)=XLTEST
        IF(ABRGE)ZS(7)=XRTEST
        IF(IXLEFT-ZS(8).LT.0)ZS(8)=IXLEFT
        ZSDEL=0
        ZS(10)=SEGPT
        IF(ABBCKL)GO TO 335
        ZS(10)=ZS(9)
        ZS(9)=SEGPT
        GO TO 335
C       MAKE A  ONE ELEMENT BOX.
331     ZS(1)=1
        ZS(2)=XLEFT
        ZS(3)=XRIGHT
        IF(ADJNEW)ZS(4)=ZLEFT
        IF(.NOT.ADJNEW)ZS(4)=ZRIGHT
        IF(ADJNEW)ZS(5)=ZRIGHT
        IF(.NOT.ADJNEW)ZS(5)=ZLEFT
        ZS(6)=XLTEST
        ZS(7)=XRTEST
        ZS(8)=IXLEFT
        ZS(9)=SEGPT
        ZSADJ=ADJNEW
        ZSDEL=DELNEW
335     CONTINUE
        NUMREF=(NUMADD+1)/2-Q(1,13)-Q(1,19)-Q(1,20)-NUMREF
        IF(NUMREF.GT.0)Q(1,14)=Q(1,14)+NUMREF
        IF(NUMADD.GT.20)NUMADD=20
        IF(NUMADD.LE.0)NUMADD=1
        ADDS(NUMADD)=ADDS(NUMADD)+1
        IF(.NOT.DEPTH)GO TO 402
        SEGPT=NEXT
        IF(FROM.EQ.0)GO TO 301
        GO TO 304
```

```
C       INTEREGATE THE ZS BOX
350     CONTINUE
        Q(1,13)=Q(1,13)+1
        IF(ZS(1)-2.LT.0)GO TO 355
        IF(SAM(1,1)-SAM(1,2).EQ.0)PAUSE 'SINGLE'
        IF(ZS(1).EQ.2.AND.(ZS(8).GE.SAM(1,2)))GO TO 400
C       SUBDIVISION NECESSARY.
        Q(1,4)=Q(1,4)+1
        IF(SEGOUT.EQ.0)GO TO 351
        SEG(SEGLO+11)=SEGACT
        SEGACT=SEGOUT
        Q(1,20)=Q(1,20)+1
        Q(1,19)=Q(1,19)+1
351     IF(ZS(8)-SAM(1,2).LT.0)GO TO 353
C       SUBDIVIDE IN THE MIDDLE.
        Q(1,12)=Q(1,12)+1
        SAM(1,2)=(SAM(1,1)+SAM(1,2))/2
        GO TO 299
C       SUBDIVIDE AT IXLEFT.
353     SAM(1,2)=ZS(8)
        GO TO 299
```

```
C       OUTPUT SEGMENTS.
355     IF(ZS(1).GT.0)GO TO 358                                          85
        SAM(2,1)=0
        Q(1,8)=Q(1,8)+1
356     XEND=SAM(1,2)
        POLYPT=0
        NEXTGO=1
        GO TO 368
358     CALL LDLPT(XEND,ZS(6))
        IF(XEND.EQ.SAM(1,1))GO TO 360
        POLYPT=0
        NEXTGO=2
        GO TO 363
360     CALL LDLPT(XEND,ZS(7))
        POLYPT=ZS(9)
        CALL LDLPT(POLYPT,SEG(POLYPT+1))
        CALL LDLPT(XTEMP,ZS(6))
        PQ(9)=PQ(9)+XEND-XTEMP+1
        NEXTGO=3
        GO TO 368
362     IF(XEND.EQ.SAM(1,2))GO TO 376
        GO TO 356
364     POLYPT=ZS(9)
        CALL LDLPT(POLYPT,SEG(POLYPT+1))
        PQ(9)=PQ(9)+SAM(1,2)-SAM(1,1)+1
        NEXTGO=4
        GO TO 368
365     XEND=SAM(1,2)
        POLYPT=ZS(10)
        CALL LDLPT(POLYPT,SEG(POLYPT+1))
        NEXTGO=1
        IF(FM.EQ.0)GO TO 368
        SAM(2,1)=IX
        SAM(3,1)=SEGPT+12
C       OUTPUT A SPECIFIC SEGMENT.
368     IF(SEGCNT.EQ.0)GO TO 372
        IF(POLYPT.NE.PRESEG)GO TO 372
        SAM(2,1)=0
        Q(1,8)=Q(1,8)+1
        GO TO 374
372     SEGCNT=SEGCNT+1
        Q(1,10)=Q(1,10)+1
        PRESEG=POLYPT
374     VISSEG(SEGCNT)=XEND
        CALL STLPT(POLYPT,VISSEG(SEGCNT))
        IF(SAM(2,1).EQ.0)GO TO 3755
C       STORE A SAMPLE POINT.
        Q(1,3)=Q(1,3)+1
        IF(SAM2S.NE.0)GO TO 375
        SAM2S=SAM(3,1)
        SAM2X=SAM(2,1)
        SAM2LX=SAM(2,1)
        SAM2L=SAM(3,1)
        GO TO 3755
375     IF(SAM(2,1).LE.SAM2LX)GO TO 3755
        SAM2LX=SAM(2,1)
        CALL STRPT(SAM(3,1),SEG(SAM2L))
        CALL STLPT(SAM(2,1),SEG(SAM2L))
        SAM2L=SAM(3,1)
3755    SAM(2,1)=0
        GO TO (376,360,362,365),NEXTGO
376     IF(SAM(1,2).EQ.FRAMEX)GO TO 498
        GO TO 281
```

```
C       INTERSECTING PLANES CASE.
400     DEPTH=.FALSE.                                                    86
        FM=0
        ZS(1)=0
        Q(1,11)=Q(1,11)+1
        SEGPT=ZS(9)
        NEXT=-1
401     XLEFT=SEG(SEGPT+3)
        XRIGHT=SEG(SEGPT+5)
        ZLEFT=SEG(SEGPT+7)
        ZRIGHT=SEG(SEGPT+9)
        NUMREF=-Q(1,13)-Q(1,19)-Q(1,20)
        Q(1,19)=Q(1,19)+1
        GO TO 317
402     NEXT=NEXT+1
        SEGPT=ZS(10)
        IF(NEXT.EQ.0)GO TO 401
        DEPTH=.TRUE.
        XXTEST=XAMXL
        CALL LDLPT(XEND,XXTEST)
        Q(1,19)=Q(1,19)+2
        IF(IY.EQ.FRAMEY)GO TO 364
        SEGSAM=ZS(10)
        CALL STLPT(ZS(9),SEGSAM)
        CALL LDLPT(SEGPT,IMPLST)
        PREV=0
4010    IF(SEGPT.EQ.0)GO TO 4030
        Q(1,19)=Q(1,19)+1
        NEXT=SEG(SEGPT)
        IF(SEGSAM.NE.SEG(SEGPT+1)) GO TO 4020
        IF(PREV.EQ.0)CALL STLPT(NEXT,IMPLST)
        IF(PREV.NE.0)SEG(PREV)=NEXT
        SEG(SEGPT+4)=XXTEST-SEG(SEGPT+3)
        SEG(SEGPT+3)=XXTEST
        CALL LDLPT(IX,SEG(SEGPT+3)+SEG(SEGPT+4))
        IF(IX.LE.0.OR.IX.GT.FRAMEX)GO TO 4040
        FM=1
        GO TO 3091
4020    PREV=SEGPT
        SEGPT=NEXT
        GO TO 4010
4030    IF(IY.EQ.FRAMEY-1)GO TO 364
        CALL GETBLK(SEGPT)
        Q(1,23)=Q(1,23)+1
        PQ(1)=PQ(1)+1
        PQ(3)=PQ(3)+1
        IF(PQ(3).GT.PQ(2))PQ(2)=PQ(3)
        SEG(SEGPT+1)=SEGSAM
        SEG(SEGPT+2)=IY-FRAMEY+1
        SEG(SEGPT+11)=-1
        SEG(SEGPT+3)=XXTEST
        CALL LDRPT(SEG(SEGPT),IMPLST)
        CALL STRPT(SEGPT,IMPLST)
        GO TO 364
4040    CALL RETBLK(SEGPT)
        PQ(3)=PQ(3)-1
        Q(1,24)=Q(1,24)+1
        GO TO 364
```

```
498     CONTINUE
        DO 499 I=1,QL
        Q(10,I)=Q(10,I)+Q(1,I)
        Q(3,I)=Q(1,I)-Q(3,I)
        IF(Q(2,I).LT.Q(3,I))Q(2,I)=Q(3,I)
        Q(8,I)=Q(1,I)-Q(8,I)
499     IF(Q(7,I).LT.Q(8,I))Q(7,I)=Q(8,I)
        IF(IY.GT.FRAMEY)GO TO 500
        IF(PIX.NE.0)CALL SHOW
        IF(SEGCNT.NE.1)PQ(10)=PQ(10)+1
        GO TO 204
500     CONTINUE
        DO 501 I=1,QL
        Q(3,I)=Q(1,I)/FRAMEY
        Q(6,I)=10.**9/(30.*Q(1,I))
501     Q(8,I)=Q(1,I)/PQ(11)
        DO 502 I=13,QL
        PQ(11)=PQ(11)+Q(9,I)
502     PQ(13)=PQ(13)+Q(10,I)
        PQ(12)=10.**9/(30.*PQ(11))
        PQ(14)=10.**9/(30.*PQ(13))
        PQ(15)=PQ(11)+PQ(13)
        PQ(16)=10.**9/(30.*PQ(15))
        IF(STAT.EQ.0)RETURN
        TYPE 5002,(J,ADDS(J),J=1,20)
        TYPE 5001,(J,PQ(J),J=1,PQL)
        TYPE 5000,(J,(Q(I,J),I=1,10),J=1,QL)
        RETURN
5001    FORMAT(' PQ(',I2,')=',I6,/)
5000    FORMAT(' Q(X,',I2,')=',I6,4I4,1X,I9,2I4,2I6,/)
5002    FORMAT(' ADDS',/,5(' (',I2,')=',I6),/)
        END
```

APPENDIX II

STATISTICS OF OBJECTS AND ALGORITHMS

At the beginning of each set of statistics for a particular program there is a description of each of the counters. Following each description, a set of statistics for each of the ten test objects is compiled.

For a copy of the complete listing write to:

Computer Science Communications
3160 MEB
University of Utah
Salt Lake City, Utah 84112